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Alexander Polynomials as Isotopy Invariants, I

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Introduction

As an isotopy invariant J. W. Alexander [1] introduced polynomials of knots, so-called the Alexander polynomials $\Delta(t)$ of knots. Recently R. H. Fox generalized this notion to the case of links and the polynomials $\Delta(t_1, \cdots, t_\mu)$ of links with multiplicity μ are called again the Alexander polynomials of links. The relation between the Alexander polynomials $\Delta(t)$ and $\Delta(t_1, \cdots, t_\mu)$ to the groups of knots and links, i. e. the fundamental groups of the complementary domain of knots and links, is studied by R. H. Fox [3] [4] by the use of his free differential calculus.

The notion of the Alexander polynomials is naturally extended to the more general cases. But the way of the extension does not seem to be unique in the method and in the subject. In this paper we shall treat the case of n-dimensional cycles $K^n (n \geqq 1)$ with integral coefficients in the $(n+2)$-dimensional sphere S^{n+2}. Of course we shall study them from the semi-linear stand point of view.

In §1 we define the Alexander polynomials of K^n in S^{n+2} by the use of free differential calculus. It should be remarked that according to R. H. Fox [4] not only one Alexander polynomial but the sequences of the Alexander polynomials are defined. In fact we have two sequences of the Alexander polynomials i. e. $\Delta^{(d)}(t_1, \cdots, t_\mu)$ and $\Delta^{(d)}(t)$. In §2 we give a presentation of the fundamental group of $S^{n+2} - |K^n|$[1] and from this we are led to the Alexander polynomials. In §3 a theorem of $\Delta^{(1)}(t_1, \cdots, t_\mu)$ is proved, which is similar to that of G. Torres [7] for the case of links. In §4 we treat briefly the general cases of theorems of E. Artin [2] and H. Seifert [6]. In §5 we shall give an example of a linear graph, which will seem to be of interest to some readers.

§1.

1. Let K^n be an n-dimensional complex with integral coeffecients in the $(n+2)$-dimensional sphere $S^{n+2} (n \geqq 1)$. Further suppose that K^n is

1) K^n is a complex with integral coefficients and $|K^n|$ is a polyhedron.