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# *The Theory of Construction of Finite Semigroups III. Finite Unipotent Semigroups.*

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As defined in the previous paper [4], we mean by a unipotent semigroup a semigroup which has unique idempotent, and in particular by a $z$-semigroup a unipotent semigroup whose unique idempotent is a zero 0, i.e. $0x=x0=0$ for all $x$; and a unipotent semigroup which contains a non-trivial group as a proper subsemigroup is called a unipotent semigroup with group.

As a special case of [2], we see that the study of finite unipotent semigroups with group is reduced to that of finite $z$-semigroups. But, as far as finite $z$-semigroups are concerned, the complete theory has not yet been established, although it has been done partly in [1], [4]. In § 1 we shall investigate the structure of finite $z$-semigroups by defining a new ordering, so that some results in the previous paper [1] will be explained more easily here. In § 2, we shall construct finite $z$-semigroups by the decompositions of certain finite free $z$-semigroups, and finally in § 3 we shall complete the construction theory of a unipotent semigroup with group to complement the results in the previous paper.

## § 1. Fundamental Properties of $z$-Semigroups.

$S$ denotes a finite $z$-semigroup. It is easily shown that a subsemigroup of $S$ is a finite $z$-semigroup and the homomorphic image of $S$ is also a finite $z$-semigroup.

**1. Partial Ordering.** Let $a$ and $b$ be elements of $S$. The element $a$ is called a multiple of the element $b$ if one of the following four equalities holds:

(1. 1) $\quad a=bx$ for some $x\in S$.

(1. 2) $\quad a=yb$ for some $y\in S$.

(1. 3) $\quad a=zbu$ for some $z,\ u\in S$.

**Lemma 1. 1.** *Every non-zero element of a finite $z$-semigroup $S$ is never a multiple of itself.*