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Osaka Math. J.
10 (1958), 43-52

On Knots and Periodic Transformations[1)]

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Introduction

Let T be a homeomorphism of the 2-sphere S^2 onto itself. If T is regular[2)] except at a finite number of points, then it is proved by B. v. Kerékjártó [11] that T is topologically equivalent to a linear transformation of complex numbers. Now let T be a homeomorphism of the 3-sphere S^3 onto itself. If T is regular except at a finite number of points, then it is known[3)] that the number of points at which T is not regular is at most two. Furthermore it is also known[4)] that if T is regular except at just two points, then T is topologically equivalent to the dilatation of S^3. Let T be sense preserving and regular except at just one point. Then whether or not T is equivalent to the translation of S^3 is not proved yet[5)]. Now let T be regular at every point of S^3. In general, in this case, T can be more complicated[6)] and there remain difficult problems[7)].

In this paper we shall be concerned with sense preserving periodic transformations of S^3 onto itself, which is a special case of regular transformations of S^3. Furthermore suppose that T is different from the identity and has at least one fixed point. Then it has been shown by P. A. Smith [19] that the set F of all fixed points of T is a simple closed curve. It is proved by D. Montgomery and L. Zippin [13] that generally T is not equivalent to the rotation of S^3 about F. It will naturally be conjectured[8)] that if T is semilinear, then T is equivalent to the rotation of S^3. In this case F is, of course, a polygonal simple

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1) A part of this paper was published in [12]. See also the footnote 11).

2) A homeomorphism T of a metric space X onto itself is called regular at $p \in X$, if for each $\varepsilon > 0$ there exists $\delta > 0$ such that if $d(p, x) < \delta$, then $d(T^n(p), T^n(x)) < \varepsilon$ for every integer n.

3) See T. Homma and S. Kinoshita [9].

4) See T. Homma and S. Kinoshita [8] [9].

5) See also H. Terasaka [21].

6) See R. H. Bing [3] D. Montgomery and L. Zippin [13].

7) See, for instance, [4] Problem 40.

8) See D. Montgomery and H. Samelson [14].