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Kernel Functions of Diffusion Equations (I)

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Let D be an open bounded set in a d-dimensional Euclidean space $\mathcal{E}$.

By Δ we understand the Laplacian with respect to given coordinates. Consider the diffusion equation

$$\frac{\partial U}{\partial t} = \Delta U \tag{1}$$

on D. By a *kernel function* $K(x, y; t)$ we understand a function on $\mathcal{E}\times\mathcal{E}\times[0, \infty)$ satisfying following properties:

(i) $K(x, y; t)=0$ when either x or y is on the boundary of D, if $K(x, y; t)$ is continuous on boundary for a fixed t, or the boundary ∂D is smooth.

(ii) For a fixed y

$$\frac{\partial}{\partial t} K(x, y; t) = \Delta_x K(x, y; t) \tag{2}$$

where Δ_x is understood as the Laplacian on the variable x.

The purpose of this paper is to give a new way of constructing the kernel function on D which coincides with the Green's function when ∂D, the boundary of D is smooth.

In preparations we shall define some notations. Coordinates of points $x, y, \cdots$ on $\mathcal{E}$ will be written x^i; $1\leq i\leq d$, y^j; $1\leq j\leq d$, etc. The euclidean distance between two points x and y is denoted by

$$|x-y| = (\sum_i^d (x^i-y^i)^2)^{1/2} . \tag{3}$$

Let

$$E_t(x, y) = (2\sqrt{\pi t})^{-d} \exp(-(4t)^{-1}|x-y|^2) . \tag{4}$$

Lemma 1. *Take a point x on D. Let $S(h)$ be a solid sphere around*

(1), see (15) and (8)
(2), see (8)