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On Cauchy Problem for Linear Partial Differential Equations with Variable Coefficients

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1. Introduction.

This paper is concerned with Cauchy problem for the general system of operators

$$\frac{\partial}{\partial t}U-AU, \tag{1.1}$$

where A is an (m, m)-matrix of differential operators of arbitrary order p_{ij} independent of $\frac{\partial}{\partial t}$ such that

$$\left.\begin{aligned} &A=(a_{ij}) \\ &a_{ij}=\sum_{\lambda_1+\cdots+\lambda_N=0}^{p_{ij}} a_{ij}^{(\lambda_1\cdots\lambda_N)} \frac{\partial^{\lambda_1+\cdots+\lambda_N}}{\partial x_1^{\lambda_1}\cdots\partial x_N^{\lambda_N}} \qquad (i, j=1, 2, \cdots, m)\,. \end{aligned}\right\} \tag{2.1}$$

The coefficients $a_{ij}^{(\lambda_1\cdots\lambda_N)}$ may depend on the time variable $t\in R_t^1$ as well as on the space variables $x=(x_1, x_2, \cdots, x_N)\in R_x^N$.

We shall prove in this note that this problem has a unique solution in a certain Hilbert space under the general condition about A: the hermitian part of A is semi-bounded in the sense of the norm induced by other hermitian matrices B of operators. This is a generalization of Leray's condition [7] which is used to solve Cauchy problem for regular hyperbolic equations. We owe our proof of this assertion in my former paper [12] essentially to Yosida's method on semigroup [13], but in the present note we prove this by another direct method using the duality of Hilbert spaces of functions defined on the product space $R_t^1\times R_x^N$. The idea originates from Nagumo, who considered more general abstract form of our theorem [10].

In Section 2 we consider a general uniformly strongly elliptic operator and introduce Hilbert spaces used in the later sections. In Section 3 we give our main theorem which is applicable to reversible systems which contain hyperbolic systems in the sense of Leray and to para-