

The Theory of Construction of Finite Semigroups II

Compositions of Semigroups, and Finite s-Decomposable Semigroups

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§ 0. Introduction.

The purpose of the present paper is to investigate construction of finite semilattices and compositions of semigroups which will play an important part in the theory of construction of finite s-decomposable semigroups. The theory of compositions of special semigroups is already included in the result obtained by Clifford [1].

A semilattice is the synonym of a commutative idempotent semigroup i.e. the multiplication system T satisfying

$$(\sigma\tau)\rho = \sigma(\tau\rho), \quad \sigma\tau = \tau\sigma, \quad \sigma^2 = \sigma$$

for all $\sigma, \tau, \rho \in T$. We have known that a semigroup S is decomposed to a semilattice T , that is to say,¹⁾

$$(0.1) \quad S = \sum_{\tau \in T} S_{\tau}, \quad S_{\tau}^2 \subset S_{\tau}, \quad S_{\tau} S_{\sigma} \subset S_{\tau\sigma} = S_{\sigma\tau}$$

where this s-decomposition of S is greatest. (Cf. [3]) The study of a semilattice is indispensable for the theory of construction of a semigroup. In this paper we shall restrict ourselves to finite semilattices and we

1) \sum denotes the set union.