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The Theory of Construction of Finite Semigroups II

Compositions of Semigroups, and Finite s-Decomposable Semigroups

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Contents

§ 0. Introduction.

The purpose of the present paper is to investigate construction of finite semilattices and compositions of semigroups which will play an important part in the theory of construction of finite s-decomposable semigroups. The theory of compositions of special semigroups is already included in the result obtained by Clifford [1].

A semilattice is the synonym of a commutative idempotent semigroup i.e. the multiplication system T satisfying

$$(\sigma\tau)\rho=\sigma(\tau\rho)\ ,\quad \sigma\tau=\tau\sigma\ ,\quad \sigma^2=\sigma$$

for all σ, τ, $\rho\in T$. We have known that a semigroup S is decomposed to a semilattice T, that is to say,[1]

$$(0.1)\qquad S=\sum_{\tau\in T}S_\tau\ ,\quad S_\tau{}^2\subset S_\tau\ ,\quad S_\tau S_\sigma\subset S_{\tau\sigma}=S_{\sigma\tau}$$

where this s-decomposition of S is greatest. (Cf. [3]) The study of a semilattice is indispensable for the theory of construction of a semigroup. In this paper we shall restrict ourselves to finite semilattices and we

1) $\sum$ denotes the set union.