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On Perron's Method for the Semi-Linear Hyperbolic System of Partial Differential Equations in Two Independent Variables

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We use the notation $\partial_x u$ for $\frac{\partial u}{\partial x}$, $\partial^2_{xy}u$ for $\frac{\partial^2 u}{\partial x \partial y}$, and write u for u_1, u_2 $\cdots$, u_k, $f(x, y, u)$ for $f(x, y, u_1, u_2, \cdots, u_k)$. $f(x, y)$ is said to be of C^1 class in a region D if $f(x, y)$ and all its first partial derivatives are continuous in D. In this note we shall consider the system of partial differential equations

$$(1) \qquad \partial_x u_i = \lambda_i(x, y)\partial_y u_i + f_i(x, y, u) \quad (i = 1, 2, \cdots, k)$$

where variables and functions are all real valued.

O. Perron[1] had discussed the Cauchy problem for the system of equations (1) under the condition that $\lambda_i, f_i, \partial_y\lambda_i, \partial_y f_i, \partial_{u_\mu} f_i, \partial^2_{yu_\mu} f_i, \partial^2_{u_\mu u_\nu} f_i (i, \mu, \nu = 1, \cdots, k)$ exist and are continuous in some region respectively.

The purpose of this note is to give such an elementary proof for the existence of solution of (1) as by Perron but under weaker assumption. We assume only the continuity of the first derivatives of λ_i, f_i except for $\partial_x\lambda_i$, $\partial_x f_i$ while the proof goes merely by a modification of Perron's method.

Recently A. Douglis[2] proved the existence of the solution of equations ot much more general type where is assumed only the continuity of the first derivatives of the functions in the form of equations. Our result is only a special case of Douglis' theorem, but it may be not insignificant to give an essentially simpler proof for this case.

1. As in Perron's theorem the proof of our theorem is also based on the following

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1) "Über Existenz und Nichtexistenz von Integralen partieller Differentialgleichungssysteme in reellen Gebieten". Math. Zeit. **27** 549–564 (1928).

2) "Some existence theorems for hyperbolic systems of partial differential equations in two independent variables". Commun. on Pure & Appl. Math. **5** (1952), 119–154. See also: K. O. Friedrichs: "Nonlinear hyperbolic differential equations for functions of two independent variables". Amer. Jour. of Math. **70** (1948), 558–589.