Osaka Mathematical Journal
Vol. 6, No. 1, December, 1954.

On Multiple Distributions

By Tadashige ISHIHARA

In the theory of quantumn wave fields, there appears a distribution called "invariant Δ-function" which gives the commutation relation between fields quantities. This Δ-function is not a function but a distribution and is considered to be defined by the wave equation $(\Box-\kappa)\cdot\Delta=0$ with initial conditions $\Delta(x, 0)=0$, $\partial\Delta/\partial t(x, 0)=-\delta_0(x)$ (c.f. J. Schwinger ([9]), W. Pauli ([10])). Concerning this sorts of equations, we consider generally here about an equation of evolution in the sense of distribution.

L. Schwartz treats this problem ([3]). He considers distributions $U_x(t)\in\mathfrak{D}'(x)$ on the spacial variables $(x_1, \dots, x_n)$ where the time variable t is a parameter. For the simplicity we call hereafter this sort of distribution a *parametric distribution* and call a distribution on the space $(x_1, \dots, x_n, t)$ a *proper distribution.* He discusses mainly parametric distribution and parametric equation of evolution. Concerning the proper one L. Schwartz refers (§16) that a parmeteric distribution can be considered to be a proper distribution and also refers to a proper distributional equation. But the relation between parametric and proper distribution and the relation between parametric and proper distributional equation is not treated in detail. In this paper we start from proper distribution conversely and researches in what case it can be considered as parametric one and researches in what case a proper equation can correspond to a parametric equation.

To give a clarification of these relations we introduce the notation of multiple distributions defined in §3, and research (§3, §4) several properties of multiple distributions.

A parametric distribution ($\in\mathfrak{D}'(x)$) is a multiple distribution of a distribution ($\in\mathfrak{D}'(x, t)$) and the special distribution ($\in\mathfrak{D}'(t)$). In §5 we consider this special case and study relations between proper distribution and parametric continuous or parametric continuously differentiable distribution. As an example of applications we discuss in §6 relations between two sorts of equations.

The invariant Δ-function mentioned at the top will be clarified in the sense of the one in the proper distributional equation, and since its corresponding parametric equation can be solved, we obtaine the