Osaka Mathematical Journal
Vol. 5, No. 1, June, 1953

Boolean Algebras and Fields of Sets

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An arbitrary Boolean algebra is isomorphic with a field of sets.[1] However, a σ-complete Boolean algebra can be no isomorph of a σ-additive field of sets: for example, the complete quotient algebra B/I where B is the family of all Borel sets of the set $[0,1]$ and I is the family of all elements of B which are of Lebesgue measure 0. In general, such a problem of the representation of an $\mathfrak{n}$-complete Boolean algebra as an $\mathfrak{n}$-additive field of sets, has been studied by a number of authors.[2]

In this paper, in relation to such a problems, we shall chiefly investigate an arbitrary Boolean algebra which is not always complete, in connection with the structure of a field of sets on which it is represented or with the existence of special measures on it. In order to investigate such a problem as clearly as possible, we introduce in §1 the conception of a ramification set and in §2 we consider a representation of a Boolean algebra on a field of sets by using ramification sets in it. The results given in §3 contain the fact that the problem already posed by A. Horn-A. Tarski in their paper [1], i.e. the problem whether for an arbitrary Boolean algebra it is atomic if and only if it is distributive in the wider sense, can be answered in the positive.

Let us notice here that it is entirely due to Theorems 1.2 and 1.4 that the theorems in §3 and §4 hold without any condition of completeness properties of a Boolean algebra.[3]

§1. Ramification set.

Throughout the present paper, the symbol A designates a Boolean algebra. In this section, we shall introduce the conception of a

1) See M. H. Stone [1], [2].

2) Among the authors, we may mention A. Tarski [1]-[5], A. Horn — A. Tarski [1], L. H. Loomis [1], [2], R. Sikorski [1], [2], L. Rieger [1] etc.

3) For notions usually used in lattice theory and for theorems, we refer to G. Birkhoff [1].