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Potential Theory and its Applications, II.

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In the potential theory the Dirichlet Problem is a central one, and this has been discussed by many authors on an abstract Riemann surface. R. Nevanlinna[1] studied it in his paper under the following conditions:

1) *The Riemann surface R is the compact subsurface of another Riemann surface $\underline{R}$.*

2) *The transfinite kernel of $\underline{R}-R$ is non empty.*

On the other hand M. Ohtsuka[2] studied under the following conditions :

1) *The projection of the Riemann surface R on another Riemann surface R is compact.*

2) *When $\underline{R}$ is a closed surface of genus zero or one, $\underline{R}-R$ contains at least three or one point respectively.*

And more precise investigation is done by him under the condition that the connectivity is finite.

But now we shall study also this problem for a non compact surface. This study is incomplete in many points as will be seen in the following. This idea owes to the paper of Brelot[3] and is intimate with those of M. Bader[4] or P. Parreu[5] rather than of R. Nevanlinna or M. Ohtsuka.

1. Let F be an abstract Riemann surface, then it is well known that there exists another non prolongable Riemann surface $\widetilde{F}$ containing the former in it. If F has finite number of genus, then $\widetilde{F}$ is a closed Riemann surface of the same genus, but if $\widetilde{F}$ has an infinite number of genus, then $\widetilde{F}$ will be an open Riemann surface. There occur two cases: either $\widetilde{F}$ is a zero-boundary or a positive-boundary Rieman sur-

1) R. Nevanlinna: Über die Lösbarkeit des Dirichletschen Problemes für eine Riemannsche Fläche, Nachr. Gött. 1 (1938), 181-193.

2) M. Ohtsuka: Dirichlet Problem on Riemann surface and conformal mapping, Nagoya Math. Jour., Vol. 3 (1951), 91-137.

3) M. Brelot: Le Problème de Dirichlet "ramifié", Ann. Grenoble, 22 (1946), 167-200.

4) R. Bader: La théorie du potential sur une surface de Riemann, C. R. Paris, 228 (1939), 2001-2002.

5) M. Parreu: A comportement à la frontière de la tonction de Green d'une surface de Riemann, C. R. Paris, 230 (1950), 709-711