Nagahata, Y. and Uchiyama, K.
Osaka J. Math.
41 (2004), 961–975

AN ESTIMATE OF THE SPECTRAL GAP FOR ZERO-RANGE-EXCLUSION DYNAMICS

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(Received April 14, 2003)

1. Introduction

This paper concerns the spectral gap for a Markovian particle system, which we call a zero-range-exclusion process. The process is a kind of lattice gas on $\mathbf{Z}^d$, which consists of particles carrying energy and whose transition mechanism is made up with a combination of dynamics for an exclusion process (for particles) and that for a zero-range process (for energy). It has two conserved quantities, the number of particles and the total energy, so that its hydrodynamic behavior must be of interest. Our process is reversible relative to certain product probability measures (serving as the grand-canonical Gibbs measures), but of non-gradient type. It will be proved that for the local process confined to a cube in $\mathbf{Z}^d$ of width n, the spectral gap is bounded below by Cn^{-2}, where C is independent of n but depends on the two order parameters, namely the number of particles per site and the energy per particle.

For the models whose grand-canonical Gibbs measures are product measures as in the present case the estimation of the spectral gap may be naturally reduced to establishing two things: one is a suitable estimate of the spectral gaps for the corresponding mean-field dynamics and the other is a certain inequality (sometimes called a moving-particle lemma) that compares a Dirichlet form for two-site dynamics of a distant pair (i.e., a pair of two sites that are far apart from each other) with a sum of those of nearest neighbor pairs (cf. [7], [2]). The former one can be obtained by adapting the arguments developed by Landim, Sethuraman and Varadhan in the paper [3] which establishes the uniform bound of the gap for zero-range processes; it has also been proved in a recent paper by Caputo [1] based on a somewhat different idea. The major ingredient in this paper therefore is a verification of the latter one, namely that of the moving-particle lemma for the present model, which is not so simple a matter as for zero-range or exclusion processes and causes the dependence on the order parameters of the constant C in the bound of the gap mentioned above. We shall also provide an indication of how to adapt the proof of [3] as well as a brief description of the approach in [1]. The uniform bound of the spectral gap for a model similar to the present one is obtained in [4], but the energy values and transition rates are uniformly bounded therein whereas they are unbounded in our model.

Our estimate of the gap, though not uniform with respect to the order parameters,