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A NON QUASI-INVARIANCE OF THE BROWNIAN MOTION ON LOOP GROUPS

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1. Introduction

In this paper, we will prove a non quasi-invariance of the Brownian motion on loop groups.

In [9], Fang proved an integration by parts formula for a natural gradient on path space over loop groups. His gradient is constructed on the parallel translation operator which was first introduced by Driver [7].

On the other hand, on path spaces over finite dimensional Lie groups, there is a natural constraction of the gradient based on the group translations. In this case, the integration by parts formula is computed via the quasi-invariance under the group translations of the reference measure.

And, there are many works on the quasi-invariance on path groups and loop groups over finite dimensional Lie groups: See, for example, Albeverio-Høeph-Krohn [3], Shigekawa [15], Malliavin-Malliavin [12].

On the contrary, our result shows that there is no extension of these results to the case of the path group over loop groups. If a smooth path acts on the law of the Brownian motion, the shifted measure is singular to the original measure except the case of the constant path.

The proof of this non quasi-invariance relies on two recent results.

One is the two parameter stochastic calculus on Lie groups which is developed in Driver-Srimurthy [8], Srimurthy [17]. This plays an essential role in the non quasi-invariance of the Brownian motion on path groups (Section 3). For two parameter stochastic calculus on manifolds, see also Norris [13].

The other one is the equivalence between the heat kernel measure and the pinned measure which is shown in Driver-Srimurthy [8] and Aida-Driver [1]. This theorem enables us to reduce the result in the path group case to the loop group case.

The organization of this paper is as follows. In Section 2, we fix some notations and give a proof for the key lemma (Lemma 2.2). In Section 3 and Section 4, we will prove the non quasi-invariance of the Brownian motion on path groups and loop groups, respectively.