Jin, N. and Masaoka, H.
Osaka J. Math.
41 (2004), 277–293

KURAMOCHI BOUNDARY AND HARMONIC FUNCTIONS WITH FINITE DIRICHLET INTEGRALS ON UNLIMITED COVERING SURFACES

Dedicated to Professor Hiroki Sato on the occasion of his sixtieth birthday

NAONDO JIN and HIROAKI MASAOKA

(Received September 20, 2002)

0. Introduction

Let W be an open Riemann surface with the Green function and $\tilde{W}$ an m-sheeted $(1 < m < \infty)$ unlimited covering surface of W. Denote by $\pi = \pi_{\tilde{W}}$ the projection of $\tilde{W}$ onto W. Consider the Kuramochi compactification W^* (resp. $\tilde{W}^*$) of W (resp. $\tilde{W}$). Denote by $\Delta = \Delta^W$ (resp. $\tilde{\Delta} = \Delta^{\tilde{W}}$) the Kuramochi boundary of W (resp. $\tilde{W}$). We also denote by $\Delta_1 = \Delta_1^W$ (resp. $\tilde{\Delta}_1 = \Delta_1^{\tilde{W}}$) the set of all minimal points in Δ (resp. $\tilde{\Delta}$). It is known that π naturally has a unique continuous extension π^* to $\tilde{W}^*$ (see [7, 2) in Proposition 2.1]). For $\zeta \in \Delta$, we set $\tilde{\Delta}_1(\zeta) = (\pi^*)^{-1}(\zeta) \cap \tilde{\Delta}_1$. Denote by $\nu(\zeta) = \nu_{\tilde{W}}(\zeta)$ the cardinal number of $\tilde{\Delta}_1(\zeta)$. Let $HD(W)$ (resp. $HD(\tilde{W})$) be the set of harmonic functions with finite Dirichlet integrals on W (resp. $\tilde{W}$). Suppose that $HD(W)$ contains a non-constant element in the sequel. Set $HD(W)\circ\pi = \{h\circ\pi : h \in HD(W)\}$. It is easily seen that $HD(W)\circ\pi \subset HD(\tilde{W})$. Then, we give necessary and sufficient conditions for the property that $HD(\tilde{W}) = HD(W)\circ\pi$ in terms of the Kuramochi compactification as follows.

Main Theorem. *The following three conditions are equivalent.*
(i) $HD(\tilde{W}) = HD(W) \circ \pi$;
(ii) *for all* $\zeta \in \Delta_1$ *except possibly for a full-polar subset of* Δ_1, $\nu(\zeta) = 1$;
(iii) *for almost every* $\zeta \in \Delta_1$ *with respect to the harmonic measure* μ_z^W $(z \in W)$ *on* Δ, $\nu(\zeta) = 1$.

By [7] we know that $1 \leq \nu(\zeta) \leq m$. According to the above theorem the property that $HD(\tilde{W}) = HD(W) \circ \pi$ is a necessary and sufficient condition to minimize $\nu(\zeta)$ for almost every $\zeta \in \Delta_1$ with respect to the harmonic measure μ_z^W $(z \in W)$ on Δ. Thus we are interested in a necessary and sufficient condition to maximize $\nu(\zeta)$, that is, $\nu(\zeta) = m$ for almost every $\zeta \in \Delta_1$ with respect to the harmonic measure μ_z^W $(z \in W)$ on Δ. We shall give a sufficient condition for the condition that $\nu(\zeta) = m$ for almost every $\zeta \in \Delta_1$ with respect to the harmonic measure μ_z^W $(z \in W)$ on Δ in the case that