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KNOTTED KLEIN BOTTLES WITH ONLY DOUBLE POINTS

AKIKO SHIMA

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1. Introduction

If an embedded 2-sphere in 4-space $\mathbf{R}^4$ has the singular set of the projection in 3-space $\mathbf{R}^3$ consisting of double points, then the 2-sphere is ambient isotopic to a ribbon 2-sphere (see [19]). Similarly, if an embedded torus in $\mathbf{R}^4$ has the singular set of the projection in $\mathbf{R}^3$ consisting only of double points, then the torus is ambient isotopic to either a ribbon torus or a torus obtained from a symmetry-spun torus by m-fusion (see [15]). In this paper we will show a similar theorem for an embedded Klein bottle in $\mathbf{R}^4$. The following is the main results in this paper.

Theorem 1.1. *Let F be an embedded Klein bottle in $\mathbf{R}^4$. If the singular set $\Gamma^*(F)$ of the projection of F in $\mathbf{R}^3$ consists only of double points, then F is ambient isotopic to either a ribbon Klein bottle or a Klein bottle obtained from a spun Klein bottle by m-fusion.*

Corollary 1.2. *Let F be an embedded Klein bottle in $\mathbf{R}^4$. Suppose that the singular set $\Gamma^*(F)$ of the projection of F in $\mathbf{R}^3$ consists of double points, and every component of the singular set $\Gamma(F)$ on F is not homotopic to zero. If the fundamental group of the complement of F is isomorphic to $\mathbf{Z}_2$, then F is trivial, i.e., F bounds a solid Klein bottle in $\mathbf{R}^4$.*

Let F be an oriented closed surface in $\mathbf{R}^4$. Is F trivial if the fundamental group of the complement of F is isomorphic to $\mathbf{Z}$? In the topological category, the question is affirmatively soloved when if it is a 2-sphere (see [3]). In the PL or smooth category, this is an open question, it is affirmatively soloved when F is one of the following:
(i) F is a 1-fusion ribbon 2-knot ([8]).
(ii) F is a 2-sphere with four critical points ([11]).
(iii) F is a symmetry-spun torus ([17]).
(iv) F is a torus whose singular set on the torus consists only of disjoint simple closed curves with non-homotopic to zero in the torus ([15]).

All homology groups are taken with coefficients in $\mathbf{Z}$, and all submanifolds are

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