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SURFACES OF REVOLUTION WITH PERIODIC MEAN CURVATURE

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1. Introduction

The surfaces of revolution with constant mean curvature in R^3 are classified by Delaunay [2] in 1841. They are locally plane, catenoid, sphere, circular cylinder, unduloid, and nodoid up to isometries of R^3. On these 10 years, new and interesting examples of non-zero constant mean curvature surfaces are discovered. In the global study of complete surfaces with constant mean curvature, unduloids and nodoids play important role as the models of ends of such surfaces (see [5], [6]). The work by Delaunay is now revived after 150 years of his discovery.

The purpose of this paper is to study surfaces of revolution with periodic mean curvature in order to extend the theory of constant mean curvature surfaces. In general such a surface is not periodic, because the catenoid gives the counter-example. First we show the criterion for a periodic function to be the mean curvature of a periodic surface of revolution and second describe a method how to construct these periodic surfaces of revolution whose mean curvatures are periodic functions satisfying the criterion.

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2. Criterion of the periodicity

Let $C = (x(s), y(s))$, $s \in I$, be a smooth plane curve parametrized by arc length on the plane $z = 0$ of R^3. We assume that the domain of the definition I is an open interval including zero and $y(s) > 0$, $s \in I$. A surface of revolution S on I is defined

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