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SOME BASIC RESULTS ON PRO-AFFINE ALGEBRAS AND IND-AFFINE SCHEMES

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Introduction

The theory of ind-affine varieties was first introduced by Shafarevich, who then employed it to elucidate the structure of the automorphism group of the affine space. (see [3], [4].) More recently we made certain revisions on the theory and applied it to study the Jacobian Problem on the endomorphisms of the complex affine space. (see [2].)

Since these pieces of work appeared, there has not been much progress made. This state may be due, in part, to the fact that the basic theory of these ind-affine or pro-affine objects as presented by us was still *ad hoc* and was rather rudimentary. So, we have embarked on building a theory of pro-affine algebras and ind-affine schemes anew and from the ground up. The outcome of our effort is the contents of the present paper. As we worked on the material we encountered a number of subtle examples, as shown in the main text below. It would seem that these examples perhaps suggest richness and mystery that this theory holds.

We mention a piece of specific result we have of our theory: The set of all morphisms of an affine variety over a field K to another may be identified with the K-rational point set of an appropriately constructed ind-affine scheme over K. This was proven using the theory of Gröbner bases over K, and is expected to be published in the near future along with certain related results about automorphisms of the affine space.

1. Pro-affine algebras

1.1. Definitions and Notations. Throughout we work over a ground field K of any characteristic. A commutative topological K-algebra A is said to be a *pro-affine algebra* if

1. A is *complete* and *separated*.
2. A base of open neighborhoods of 0 is given by a family of *countably many* ideals $\subseteq A$.

Let $\{\mathfrak{a}_i : i \in \mathbb{N}\}$ be a countable base referred to just above. Here, as elsewhere throughout the present paper, $\mathbb{N}$ represents the set of all *nonnegative* integers. We may,