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ON A FRAMEWORK OF SCATTERING FOR DISSIPATIVE SYSTEMS

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1. Introduction

In this paper we study the existence of scattering solutions for some dissipative systems which contain elastic wave with dissipative boundary conditions in a half space of $\mathbf{R}^3$ (cf. Dermenjian-Guillot [1]). First we give a framework based on the idea of Simon [18] and apply it to elastic wave mentioned above. In applying the abstract framework, we shall use the Mellin transformation (cf. Perry [14]) as a key tool.

Let $\mathcal{H}$ be separable Hilbert space with inner $\langle\cdot,\cdot\rangle_{\mathcal{H}}$. The norm is denoted by $\|\cdot\|_{\mathcal{H}}$. Let $\{V(t)\}_{t\geqq 0}$ and $\{U_0(t)\}_{t\in\mathbf{R}}$ be a contraction semi- group in $\mathcal{H}$ and a unitary group in $\mathcal{H}_0$, respectively. We denote the generator of $V(t)$ and $U_0(t)$ by A and A_0, respectively ($V(t)=e^{-itA}$ and $U_0(t)=e^{-itA_0}$). We make the following assumptions on A and A_0.

(A1) $\sigma(A_0)=\sigma_{ac}(A_0)=\mathbf{R}$ or $[0,\infty)$.

(A2) $(A-i)^{-1}-(A_0-i)^{-1}$ defined as a form is extended to a compact operator K in $\mathcal{H}$.

(A3) There exist non-zero projection operators in $\mathcal{H}$, P_+ and P_-, such that $P_+ + P_- = I_d$ and

$$\int_0^\infty \|KU_0(t)\psi(A_0)P_+\|\,dt<\infty, \tag{A3.1}$$

$$\int_0^\infty \|K^*U_0(t)\psi(A_0)P_+\|\,dt<\infty, \tag{A3.2}$$

$$\int_0^\infty \|K^*U_0(-t)\psi(A_0)P_-\|\,dt<\infty, \tag{A3.3}$$

$$\mathrm{w}-\lim_{t\to+\infty} U_0(-t)\psi(A_0)P_-f_t=0, \tag{A3.4}$$

for each $\psi\in C_0^\infty(\mathbf{R}\backslash 0)$ and $\{f_t\}_{t\in\mathbf{R}}$ satisfying $\sup_{t\in\mathbf{R}}\|f_t\|_{\mathcal{H}}<\infty$, where $\|\cdot\|$ is the operator norm of bounded operators in $\mathcal{H}$.

(A3.1), (A3.3) and (A3.4) will imply the existence of the wave operator. It will follow from (A3.2) that the wave operator is not zero as an operator in $\mathcal{H}$. The framework of [18] is due to Enss's method [2]. In order to cheak the applicability of the framework of [18] to dissipative systems (see also Stefanov-Georgiev [20] or [15]), we