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ON HAEFLIGER'S OBSTRUCTIONS TO EMBEDDINGS AND TRANSFER MAPS

Dedicated to the memory of Professor Katsuo Kawakubo

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1. Introduction and statement of results

Throughout this article, n-manifolds mean compact differentiable (or topological) manifolds of dimension n. The (co-)homology is understood to have $\boldsymbol{Z}_2$ for coefficients.

For a manifold V, we denote by $w(V)$ and $\bar{w}(V)\,(= w(V)^{-1})$, the total Stiefel-Whitney class and the total normal Stiefel-Whitney class of V, respectively. Furthermore, we denote by $U_V \in H^{\dim V}(V \times V)$ the $\boldsymbol{Z}_2$-Thom class (or $\boldsymbol{Z}_2$-diagonal cohomology class) of V [10, p. 125]. For a (continuous) map $f\colon M^n \to N^{n+k}$ between closed manifolds M and N, we define the total Stiefel-Whitney class $w(f) = \sum_{i\geq 0} w_i(f)$ by the equation

$$w(f) = \bar{w}(M) f^* w(N).$$

For a map $f\colon M^n \to N^{n+k}$, the transfer map (or Umkehr homomorphism) $f_!\colon H^i(M) \to H^{i+k}(N)$ is defined by the commutative diagram below:

$$\begin{array}{ccc} H^i(M) & \xrightarrow{f_!} & H^{i+k}(N) \\ \cong \Big\downarrow \cap[M] & & \cong \Big\downarrow \cap[N] \\ H_{n-i}(M) & \xrightarrow{f_*} & H_{n-i}(N). \end{array}$$

Here $[V] \in H_{\dim V}(V)$ denotes the fundamental homology class of a manifold V.

Our main theorem is the following

Theorem 1.1. *For a continuous map $f\colon M^n \to N^{n+k}$ between closed topological manifolds, $U_M(1 \times w_k(f)) + (f \times f)^* U_N = 0$ if and only if $f^* f_!(a) = a w_k(f)$ for all $a \in H^*(M)$.*

The cohomology elements, appearing in this theorem, are related to the embeddability of f. A. Haefliger [7, Théorèm 5.2] proved the following