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A VARIATION ON THE GLAUBERMAN CORRESPONDENCE

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1. Introduction

Suppose that G is a finite p-solvable group, where p is a prime. Let $\mathrm{IBr}(G)$ be the set of irreducible Brauer characters of G, and let $\mathrm{IBr}_{p'}(G)$ be those $\varphi \in \mathrm{IBr}(G)$ of degree not divisible by p.

The Glauberman correspondence, in the important case where a p-group acts on a p'-group, can be viewed as a natural correspondence between $\mathrm{IBr}_{p'}(G)$ and $\mathrm{IBr}(\mathbf{N}_G(P))$, where $P \in \mathrm{Syl}_p(G)$ and G is a group with a normal p-complement. Our point in this note is to show that it is not necessary to assume that G has a normal p-complement: it suffices to assume that $\mathbf{N}_G(P)$ does.

Theorem A. *Suppose that G is p-solvable, and let $P \in \mathrm{Syl}_p(G)$. Assume that $\mathbf{N}_G(P)$ has a normal p-complement. Then for every $\varphi \in \mathrm{IBr}_{p'}(G)$, there is a unique $\varphi^* \in \mathrm{IBr}(\mathbf{N}_G(P))$ such that*

$$\varphi_{\mathbf{N}_G(P)} = e\varphi^* + p\Delta\,,$$

where e is not divisible by p and Δ is some Brauer character of $\mathbf{N}_G(P)$ or zero. Also, the map $\mathrm{IBr}_{p'}(G) \to \mathrm{IBr}(\mathbf{N}_G(P))$ given by $\varphi \mapsto \varphi^*$ is a bijection. On the other hand, if $\tau \in \mathrm{IBr}(G)$ has degree divisible by p, then

$$\tau_{\mathbf{N}_G(P)} = p\Xi\,,$$

where Ξ is some Brauer character of $\mathbf{N}_G(P)$.

Even in the case where $\mathbf{N}_G(P) = P$, Theorem A above tells us something nontrivial (although well-known): a Sylow p-subgroup P of a p-solvable group G is self-normalizing, if and only if all nontrivial irreducible Brauer characters of G have degree divisible by p.

The condition of $\mathbf{N}_G(P)$ having a normal p-complement is natural enough that can be read off from the character table of G (whenever G is p-solvable).

Theorem B. *Suppose that G is p-solvable and let $P \in \mathrm{Syl}_p(G)$. Then $\mathbf{N}_G(P)$ has a normal p-complement iff the number of p-regular classes of G of size not divis-*