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On $(k+1)$*-ad homotopy groups*

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A. L. Blakers and W. S. Massey[1] have defined triad homotopy groups and used it for the problem of absolute and relative homotopy groups. They also refered to $(k+1)$-*ad* homotopy groups. I want here to prove the exactness of sequence of $(k+1)$-*ad* homotopy groups, and apply to the simplest case. The author wishes to express his cordial thanks to Prof. A. Komatu, and Mr. J. Nagata, for their encouragement in this paper.

1. Notation and terminology.

Let X and Y be topological spaces, $A_1, \cdots, A_k$ subspaces of X, and $B_1, \cdots, B_k$ subspaces of Y.

The notation

(1. 1) $$f\colon (X;\ A_1, \cdots, A_k) \to (Y;\ B_1, \cdots, B_k)$$

means that f is a continuous function defined on X with values in Y, satisfying the condition

$$f(A_i) \subset B_i, \quad (i=1, 2, \cdots\cdots k).$$

If the sets $A_1, \cdots, A_k$ have a non-vacuous intersection, $A_1 \frown A_2 \frown \cdots \frown A_k = C \neq 0$, we call the ordered collection of spaces $(X;\ A_1, \cdots, A_k)$ a $(k+1)$-*ad.* An n-cell E^n is the set of vectors $\mathfrak{x}=(x_1, \cdots, x_n)$, where $0 \leqq x_i \leqq 1$.

The symbol $F_n^k(X;\ A_1, \cdots, A_k, p_0)(p_0 \in A_1 \frown A_2 \frown \cdots \frown A_k)$ will denote the function space of all maps

$$f\colon (E^n \times E^k) \to (X)\,,$$

such that, for all vectors $\mathfrak{x}=(x_1, \cdots, x_n) \in E^n$ and $\mathfrak{y}=(y_1, \cdots, y_k) \in E^k$,

(1. 2) $$f(x_1, \cdots, x_n, y_1, \cdots, y_k) = p_0 \quad (\text{if one of } x_i=0 \text{ or } 1, \text{ or one of } y_j=1),$$
$$f(x_1, \cdots, x_n, y_1, \cdots, y_k) \in A_i \quad (\text{if } y_i=0),$$

and we introduce the compact open topology in it.

Analogously, we use another symbol $\overline{F}_n^k\ (X;\ A_1, \cdots, A_k)$, which is the function space of all maps

$$f\colon (\dot{E}^{n+1} \times E^k) \to X\,,$$

such that

(1. 3) $$f(x_1, \cdots, x_{n+1}, y_1, \cdots, y_k) = p_0 \quad (\text{if } \mathfrak{x}=(x_1, 0 \cdots 0) \text{ or } x_i=0 \text{ or } x_i=1 \text{ or one of } y_i=1),$$