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On homotopy classification and extension

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It is the purpose of this paper to discuss the rôle of the groups $H(\Pi, n, \Pi', q, \boldsymbol{k})$ in the study of the obstruction and classification theorems for mappings of a geometric complex K into a topological space Y such that

$$\pi_i(Y)=0 \text{ for } 0\leqq i<n,\ n<i<q, \text{ and } q<i<r<2q-1,$$

along the line of Eilenberg-MacLane [3].

As is well-known, the space Y has the invariants $\boldsymbol{k}_n^{q+1}\in H^{q+1}(\pi_n, n; \pi_q)$ and $\boldsymbol{k}_q^{r+1}\in H^{r+1}(\pi_q, q; \pi_r)$.[1] In addition to these, as is shown in §6, there is an invariant $\{\boldsymbol{k}_{n\cdot q}^{r+1}\}$ which is a coset of $H^{r+1}(\pi_n, n, \pi_q, q, \boldsymbol{k}_n^{q+1}; \pi_r)$.

Let K be a geometric complex with subcomplex L and $f: K^n\cup L\rightarrow Y$ be a mapping extensible to a map $K^{q+1}\cup L\rightarrow Y$. The third obstruction to the extension of f is then a coset of $H^{r+1}(K, L; \pi_r)$. This obstruction was treated by N. Shimada and H. Uehara in some special cases [1].

Our main purpose is the expression of this coset in the general cases, and by an application we shall explain the allied extension and classification theorems in terms of our new operators $\boldsymbol{y}_\gamma$ and $\boldsymbol{y}_\tau$ which are introduced is §4. Throughout, we omit the case $n=1$.

§1. The maps $T(x_n, x_q)$.

For any (discrete) abelian groups Π, Π', any integers n, q $(1<n<q)$, and any cocycle k of $Z^{q+1}(\Pi, n; \Pi')$ we shall introduce an R-complex $K(\Pi, n, \Pi', q, k)$ which is a k-prolongation of $K(\Pi, n)$ in a sence.[2]

A p-cell of $K(\Pi, n, \Pi', q, k)$ is a pair (ϕ, ψ), where ϕ is a p-cell of $K(\Pi, n)$, and ψ is an element of $F_p(\Pi', q)$ subject to the condition;

$$\textstyle\sum_{i=0}^{q+1}(-1)^i\psi(\gamma_{q+1}^i)+k(\phi\gamma)=0 \quad \text{for any map } \gamma\in K_{q+1}(p)\,.$$

The internal product of two such p-simplices (ϕ, ψ), (ϕ', ψ') is $(\phi\circ\phi', \psi\circ\psi')$ where

$$(\phi\circ\phi')(\alpha)=\phi(\alpha)+\phi'(\alpha)\,,\quad (\psi\circ\psi')(\beta)=\psi(\beta)+\psi'(\beta)$$

for arbitrary appropriate dimensional maps α, β. And the p-simplex $(\iota_{p.n}, \iota_{p.q})$ which is a pair of the neutral elements determines the unit for this product.

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1) For the sake of brevity, we write in the following $\pi_n=\pi_n(Y)$, $\pi_q=\pi_q(Y)$, and $\pi_r=\pi_r(Y)$.
2) Refer. [2].