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ALGEBRA OF STABLE HOMOTOPY OF MOORE SPACE

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0. Introduction

Let p denote an odd prime. A Moore space $M_p^n = M(n, Z_p)$ is a simply connected space with two non-vanishing (integral) homology groups $H_0(M_p^n)=Z$ and $H_n(M_p^n)=Z_p$. The mod p cohomology structure of M_p^n is as follows: $H^0(M_p^n; Z_p)=Z_p$, $H^n(M_p^n; Z_p)=Z_p=\{e^n\}$, $H^{n+1}(M_p^n; Z_p)=Z_p=\{e^{n+1}\}$, $H^i(M_p^n; Z_p)=0$, $i\neq 0$, n, $n+1$, and $\Delta e^n = e^{n+1}$ for the mod p Bockstein operator Δ, for $n\geqq 2$.

The m-th homotopy group $\pi_m(Z_p; n, Z_p)$ of the Moore space $M(n, Z_p)$ with the coefficient group Z_p (or, briefly, *the m-th mod p homotopy group of* $M(n, Z_p)$) is the set of homotopy classes of maps $M_p^m \rightarrow M_p^n$ with the track addition (See [3]).

The set $\pi_* = \sum_i \pi_{N+i}(Z_p; N, Z_p)$ (N denotes a sufficiently large integer) of the stable homotopy groups of the Moore space $M(N, Z_p)$ with the coefficient group Z_p (i.e., the stable mod p homotopy groups of $M(N, Z_p)$) admits a ring structrue with respect to the composition. Really, it forms an algebra over the field Z_p.

In this paper, we shall investigate its structure by means of the results and the methods of Toda [10], [11], [12].

For simplicity, we shall denote $\pi_{N+i}(Z_p; N, Z_p)$ by π_i and we shall say that an element of π_i is of dimension i.

Among the elements of π_*, δ denotes the element in π_{-1} such that $\delta^* e_2^N = (-1)^N e_1^N$ for the generators $e_1^N \in H^N(M_p^{N-1}; Z_p)$and $e_2^N \in H^N(M_p^N; Z_p)$; ι denotes the class of the identity map of M_p^N; α denotes the element in $\pi_{2(p-1)}$ such that $\mathcal{P}_\alpha^1 e^{N+1} = (-1)^{N+1} e^{N+2(p-1)}$ for the generators $e^{N+1} \in H^{N+1}(M_p^N; Z_p)$ and $e^{N+k} \in H^{N+k}(M_p^{N+k}; Z_p)$, $k=2(p-1)$, where $\mathcal{P}_\alpha^1$ is the functional cohomological operation with respect to $\mathcal{P}^1$ and α; β_1 denotes the element in $\pi_{2p(p-1)-1}$ such that $\alpha\beta_1=0$ and $\mathcal{P}_{\beta_1}^p e^{N+1} = (-1)^{N+1} e^{N+2p(p-1)}$ for the generators $e^{N+1} \in H^{N+1}(M_p^N; Z_p)$ and $e^{N+l} \in H^{N+l}(M_p^{N+l-1}; Z_p)$, $l=2p(p-1)$, where $\mathcal{P}_{\beta_1}^p$ is the functional cohomological operation with respect to $\mathcal{P}^p$ and β_1; and, β_s, $1<s<p$, denote the element in $\pi_{2(sp+s-1)(p-1)-1}$