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On the homotopy groups of Stiefel manifolds

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J. H. C. Whitehead [3] gave a cellular decomposition of the Stiefel manifold $V_{k+m,\,m}$ of m-frames in Euclidian $(k+m)$-space, and he and Baratt-Peacher [1] determined the homotopy groups $\pi_{k+j}\,(V_{k+m,\,m})$ for $j=1, 2, 3$.

In the present paper, by making use of the above J. H. C. Whitehead's result and the Steenrod square, we shall gave a reduced cell complex of P^{l}_{k-1} and determine $\pi_{k+j}\,(V_{k+m,\,m})$ for $j\leqslant 5$ $(k\geqslant j+2)$.

The basic tools used are following:

i) (*Whitehead's theorem*) [3 theorem 3]. If $r<2k$, then $\pi_r\,(V_{n,\,m})=\pi_r\,(P^{l}_{k-1})$, $l=\mathrm{Min}\,(r+1, n-1)$, $k=n-m$, where P^{l}_{k-1} is a space obtained from the l-dimensionul projective space by shrinking its $(k-1)$-dimensional hyperplane to a point.

ii) (*Squaring formula*). If we denote by u^j the generator of $H^j\,(P^{l}_{k-1};\,Z_2)$, we have $Sq^i u^j=\binom{i}{j}u^{i+j}$ for $i+j\leqslant l$, where $\binom{j}{i}$ is the binomal coefficient with the usual conventions.

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1. Notations.

We shall use the following notations throughout this paper.

P^{n}_{k-1}: the space obtained from the l-dimensional projective space by shrinking its $(k-1)$-dimensional hyperplane to a point.

We denote $\pi_{n+r}\,(P^{n+k}_{n-1})$, $\pi_{n+r}\,(P^{n+k}_{n-1}, P^{n+k-1}_{n-1})$ by π^k_r, σ^k_r respectively.

Let $K=L\underset{f}{\smile}e^{n+1}$ be a complex such that e^{n+1} is attached to L by a mapping f.

A map $\bar{g}e^{n+1}$: $(E^{p+1}, \dot{E}^{p+1})\rightarrow(K, L)$ is defined as follows, where g is a map S^p to S^n; $\bar{g}e^{n+1}$ maps E^{p+1} in e^{n+1} by the suspension of g, $\bar{g}e^{n+1}|\dot{E}^{p+1}$ in K by $f\cdot g$.

Now if $f\cdot g$ is a nullhomotopic in L, we denote by ge^{n+1}: $S^{p+1}=E^{p+1}_{+}\smile E^{p+1}_{-}\rightarrow K$ the following map: $ge^{n+1}|E^{p+1}_{+}$ maps E^{p+1}_{+} in e^{n+1} by the suspension of g, and $ge^{n+1}|E^{p+1}_{-}$ is a null homotopy of $f\cdot g$.

$\{\bar{g}e^{n+1}\}_q$, $\{ge^{n+1}\}_q$ are cyclic subgroups of order q of $\pi_{p+1}(K, L)$, $\pi_{p+1}(K)$ which are generated by $\{\bar{g}e^{n+1}\}$, $\{ge^{n+1}\}$ whose representatives are $\bar{g}e^{n+1}$, ge^{n+1} respectively.

We denote the generators and these representatives of $\pi_{n+1}\,(S^n)$, $\pi_{n+2}\,(S^n)$, $\pi_{n+3}\,(S^n)$, by the same letters η, ε, ν respectively.