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On randomizations of confidence intervals

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In recent years it seems to the author that it has been devoted by some statisticians and mathematicians to rewrite a considerable part of the theory of statistics in the terms of statistical decision functions. The theory of confidence intervals is not exceptional. This leads writers of the theory to the consideration of randomized confidence intervals for the sake of the theoretical and practical treatment of confidence intervals.

The author has been informed, up to this time, of two types of expressions of randomized confidence intervals.

(1) W-type randomization; depending on every sample point x observed, we choose a random machine m_x and then select an interval by means of m_x. For example, we shoot, for every observed x, an arrow which hits in a subset S of the half plane $A=\{(u, v) \mid u \leqq v\}$ with probability $m_x(S)$. If it hits a point $a=(u, v)$, we estimate the true parameter θ to belong to the interval limited by u and v.

The readers can see such a randomization in Wald's Book (Wald [1]). We shall call this a W-type randomization.

(2) S-type randomization; some writers have used another type of randomization though it is rather vaguely defined (Stein [2]). It gives only the probability $\varphi_x(\theta)$ that confidence interval contains θ for observed x. So it does not tell us how to estimate the true parameter θ. We shall call it an S-type randomization. There is some φ_x which can not be expressed in the preceding form.

The first aim of this note is to give necessary and sufficient conditions for an S-type to be expressible as a W-type. According to them, φ_x has an expression for a W-type if and only if the total variation of $\varphi_x(\theta)$ as a function of θ does not exceed 2 for every x.

Next we consider the set $\mathfrak{F}^*$ of all φ_x whose total variation as a function of θ is exactly 2 for every x and the set $\mathfrak{M}^*$ of all m_x by which we estimate, for every x, an interval containing θ to be surely on a curve c_x in A. And we shall prove that these subclasses $\mathfrak{F}^*$ and $\mathfrak{M}^*$ are essentially complete in the class of all W-type randomizations and of all S-type randomizations respectively.

At last we shall show equivalences of these classes. These equivalences are not so clear as at the first glance when we adopt the confidence coefficient and the average length as a criterion of optimalities.