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On the Singularity of the Peturbation-Term in the Field Quantum Mechanics

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Summary

Let H be a total Hamiltonian of a system consisting of two fields. When H is divided into two parts in two ways as $H=H_1^0+H_1'=H_2^0+H_2'$, where H_1^0 and H_2^0 are unperturbed terms and H_1' and H_2' are perturbation terms, then (i) two spaces $\mathfrak{H}(H_1^0)$ and $\mathfrak{H}(H_2^0)$ which are determined by the systems of eigenvectors of H_1^0 and H_2^0 respectively, are mutually orthogonal, and (ii) the zero point energy of H_1^0 differs from that of H_2^0 by infinity. The zero point energy of the total Hamiltonian of a system in which a fixed nucleon and a real scalar meson field are interacting, amounts to $-g^2c^2/4V\cdot\sum 1/\omega_k^2$ which diverges to minus infinity. The total Hamiltonian of a system electron plus photon field has the expectation value $(H\Psi_\beta, \Psi_\beta)=c_\beta+\sum(1/2-2\pi c^2e^2l_\lambda^2/hV\omega_\lambda^3)\,h\omega_\lambda$, where Ψ_β is a certain vector normalized to 4, c_β a finite constant depending on β, and l_λ the projection on the x-axis of the polarization vector e_λ of the λ-photon. The number of Ψ_β's is enumerably infinite and they are orthogonal with one another.

1. Introduction

In the previous paper[1] we have proved that the interaction term of a system which consists of a nucleon and a complex scalar meson field has no domain in a space, each of whose vectors is a superposition of states consisting of the nucleon and a finite number of mesons.

In a similar way, we can prove that the interaction term of a system electron field plus photon field has no domain in a space, each of whose vectors is a superposition of states which consits of a finite number of electrons and photons. The proof will be given elswhere.

Thus a vector representing a state in which an electron and a photon are in respective given state, does not belong to the domain of the total Hamiltonian. Here the zero point energy of the non-interacting term is not taken into account.

The total Hamiltonian operator is usually divided into two parts, the one is the principal part H^0 and the other is the perturbation term H'. H^0 can be transformed into a diagonal form by a suitable unitary transformation and