

On topological groups.

By Shin-ichi MATSUSHITA

(received September 15, 1952)

0. **Introduction.** In this paper, we shall deal with arbitrary topological groups by means of their *Markoff-extensions*: the definition of an *Markoff-extension* is given in Section 1.

Generally speaking, though the representation theory in matric-algebras plays an important rôle in studying topological groups,¹⁾ it becomes occasionally meaning-less for some type of groups, which have no usual (**non**-trivial) representations; as well known, minimally almost periodic groups are those. However, the Markoff-extension seems to be useful for any topological groups.

Section 2 is devoted to an exposition of the relation between the representation of a topological group and those of its Markoff-extension. In Section 3, we shall concern the duality theorem of any topological groups, which we would rather call the *co-duality theorem*. Our theorem coincides with the famous one of Tannaka and Krein²⁾ in maximally almost periodic cases at all, but even if a group is minimally almost periodic, ours may remain still useful.

This duality theorem is, on the other hand, considered as the representation theorem in B-algebra, and the process from Theorem 4 to Theorem 5 gives one proof for the Tannaka-Krein's duality theorem.

The space of almost periodic functions is considered as a commutative B*-algebra. This investigation is done in Section 4.

Finally, in Section 5, we shall try the theorem of K. Iwasawa³⁾ concerning the group-rings as an interesting application of Markoff-extensions.

1. **Preliminary theorem.** We begin with the noted theorem of A. Markoff and S. Kakutani on free topological groups.⁴⁾ That is stated as follows: For any completely regular topological space Γ , there exists a free topological group F with the following properties;

-
- 1) Recently, Banach representation theory has been developed as in 13), 19), 20), etc. But in them, groups are restricted in locally compact case.
 - 2) T. Tannaka 24), and M. Krein 10).
 - 3) K. Iwasawa 5) and 6).
 - 4) A. Markoff 11) and S. Kakutani 8).