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Homological Structure of Fibre Bundles

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1. To find relations existing among the homological characters of the bundle space, of the base space, and of the fibre of a given fibre bundle is an important problem in topology.

In the preceding paper [9] the author, in connection with this problem, gave a new formulation of the so-called Leray's algorism [10] on the one hand, and generalized two theorems of Samelson concerning to homogeneous spaces to theorems of fibre bundles (see §2 below) on the other hand.

The purpose of the present paper is to give them more detailed accounts and to derive almost all theorems in our direction.[1] In part II characteristic groups and characteristic isomorphisms are defined for arbitrary set systems, and their fundamental properties are given.[2] In this form they reveal a close bearing on the theory of Morse, classifying cycles according to critical levels. Moreover they may be applied to homotopy as well as cohomotopy theories. In particular, if applied to cohomotopy theory, they give a formal answer to the classification problem of maps of an $(n+r)$-complex into an n-sphere for arbitrary r but for sufficiently large n (II, §4). In part III results of part II are applied to fibre bundles over a complex. In part IV various formulas concerning to $\cup$- and $\cap$-multiplications are given, and the theorems of Gysin [5], of Thom-Chern-Spanier [23], [3], and of Wang [24] are generalized. In part V $\circ$- and $\square$-multiplications are introduced and as application several important theorems about homological trivialness are given, some of of which[3] seem to be contained in the results announced by Hirsch [6].

2. To explain our problem we shall give here some theorems about homological triviality.

Theorem A. (*Künneth' theorem*) *If A is the product complex of two complexes B and F, the cohomology ring $H^*(A)$ is isomorphic to the Kronecker product $H^*(B)\otimes H^*(F)$ of the cohomology rings $H^*(B)$, $H^*(F)$ of B, F respectively, where the rational number field is taken as the coefficient ring.*

1) Major parts of this paper (Part II-IV) were published in Japanese in March, 1951.

2) Another abstract formulation of Leray's algorism was obtained by H. Cartan, J. Leray [30], and J. L. Koszul [8].

3) Theorem 22 and Theorem 23. In the case of homogeneous space these theorems are consequences of the results proved by Koszul [8].