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*On the Non-existence of Solution of Field Equations in Quantum Mechanics**

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Summary

The Hamiltonian operator of a system consisting of a nucleon and a scalar meson field, has no domain in the space whose vector is a superposition of states consisting of a finite number of mesons and a nucleon, in which the states of each meson and the nucleon being arbitrary.

§1. Introduction. As physicists understand, the field equations in quantum dynamics have no solution when the interaction terms are taken into account. But the rigorous proof of the non-existence of the solution seems not to have been given so tar.

In the present paper, first we determine the space in which the Hamiltonian of a system is defined, and then prove that the Hamiltonian operator has no domain in its subspace whose state vector is a superposition of states consisting of a finite number of particles, in which the state of each particle being arbitrary. For the sake of the simplicity of the treatment, we take, as an example, a system consisting of a nucleon and a scalar meson field.

In the course of the treatment, we assume that the wave functions of the nucleon and the scalar meson field are periodical with respect to a unit cube in the coordinate space. This assumption is not satisfactory in the scope of the relativity. The relativistically complete treatment will be made in another place.

§2. Determination of the space. First we state the results obtained by v. Neumann[1], in a form suitable for our purpose.

Let I be a set of indices α, whose number is enumerably infinite. $\mathfrak{H}_\alpha, \alpha \in I$ is a sequence of Hilbert spaces.

Definition 1. $z_\alpha, \alpha \in I$ and a are arbitrary complex numbers. Then the product $\Pi_{\alpha\in I} z_\alpha$ is convergent and its value is a when the following condition is satisfied. Let δ be an arbitrary positive number, then, corresponding to this δ, a finite set $I_0 = I_0(\delta) \subset I$ of α's can be determined in such a way that the difference

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