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On Conditions in order that two Uniform Spaces are uniformly homeomorphic

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The topology of a compact space is, as well known, characterized by the lattice of its closed basis.

The purpose of this paper is to establish analogous theories in the case of complete uniform spaces, i.e. we characterize the uniform topology of a uniform space by the lattice of its uniform basis. Obviously, it is impossible to characterize the uniform topology of a complete space by the lattice of an arbitrary uniform basis[1], for every metric space has the same lattice of uniform basis $\{\mathfrak{M}_1, \mathfrak{M}_2, \mathfrak{M}_3, \ldots\}$.

For example, let us consider the reason why the ring of all continuous functions characterizes the topology of a compact space. Then we shall see that it is the reason that for an arbitrary point a and its arbitrary neighbourhood U, there exists a continuous function f such that $f(a)=0$, $f(x)=1$ $(x \in U^c)$. Similarly if we assume the existence of uniform coverings with some local properties in the lattice of uniform basis, then we can characterize the uniform topology of a complete uniform space by such a lattice.

We concern ourselves with the lattice $L(R)$ of uniform basis of a complete uniform space R, satisfying the following conditions,

1) *if* $\mathfrak{M}_x, \mathfrak{M}_y \in L(R)$, *then* $\mathfrak{M}_x \times \mathfrak{M}_y \in L(R)$,

2) *if* $\mathfrak{M} \ni L(R)$, *then for an arbitrary* $M \in \mathfrak{M}$ *there exists* $\mathfrak{M}(M)$ *in* $L(R)$ *such that* i) $M \notin \mathfrak{M}(M)$;

ii) $\mathfrak{M} \ni M' \not\supset M$ *implies* $M' \in \mathfrak{M}(M)$,

3) *for an arbitrary point* $a \in R$ *and its arbitrary neighbourhood* $U(a)$, *there exists* $\mathfrak{M}'(U(a))$ *in* $L(R)$ *such that* $S(a, \mathfrak{M}'(U(a))) \subset U(a)$[2], *for a uniform covering* $\mathfrak{M}'(a)$ *defined for* a, $a \notin M \in \mathfrak{M}'(U(a))$ *implies* $M \notin \mathfrak{M}'(a)$,

or 2′) *if* $\mathfrak{M} \in L(R)$, *then for an arbitrary* $M \in \mathfrak{M}$, *there exists* $\mathfrak{M}(M)$ *in* $L(R)$ *such that* i) $M \notin \mathfrak{M}(M)$;

ii) $M' \in \mathfrak{M}$, $\bar{M}' \not\supset M$ *imply* $M' \in \mathfrak{M}(M)$,

3′) *for an arbitrary point* $a \in R$ *and its arbitrary neighbourhood* $U(a)$,

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1) If $L(R)$ is a set of open uniform coverings of R, and if for every uniform covering $\mathfrak{U}$ of R, there exists some element $\mathfrak{M}$ of $L(R)$ such that $\mathfrak{M} < \mathfrak{U}$, then we call $L(R)$ a uniform basis of R.

2) $S(a, \mathfrak{M}) = \cup \{M \mid a \in M \in \mathfrak{M}\}$.