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Some Relations in Homotopy Groups of Spheres

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Introduction

It is well known that the suspension (*Einhängung*) homomorphism $E: \pi_n(S^r) \to \pi_{n+1}(S^{r+1})$ is isomorshism if $n<2r-1$ [3] [1].* In recent years, G. W. Whitehead has shown that the kernel of the suspension homomorphism E is the subgroup generated by whitehead product, if $n=2r-1$ [9, §7].

In this paper we shall calculate some special whitehead products, and indicate some non-trivial suspension homomorphisms. For example, in cases where $n=r+4\,(r=2,4,5)$ and $n=r+5\,(r=2,4,5,6)$ E is not isomorphic, and also we obtain non-zero elements of $\pi_{4n+10}(S^{2n+4})$ and $\pi_{4n+22}(S^{2n+8})$ $(n=0, 1, \dots.)$, whose suspension vanish.

1. Notations

We shall use the notations analogous to those of G. W. Withehead [9, §1]. Define

$$S^n=\{(x_1, \dots, x_{n+1})\,|\,\sum x_i^2=1\},$$
$$E_+^n=\{x\in S^n\,|\,x_{n+1}\geqq 0\}, \qquad E_-^n=\{x\in S^n\,|\,x_{n+1}\leqq 0\},$$
$$I^n=\{(x_1, \dots, x_n)\,|\,-1\leqq x_i\leqq 1\},$$
$$\dot I^n=\{(x_1, \dots, x_n)\,|\,\Pi\,(1-x_i^2)=1\},$$
$$J_+^n=\{x\in \dot I^{n+1}\,|\,x_{n+1}\geqq 0\}, \qquad J_-^n=\{x\in \dot I^{n+1}\,|\,x_{n+1}\leqq 0\},$$
$$y_*=(1, 0, \dots, 0), \qquad 0=(0, \dots, 0),$$
$$S^n\vee S^n=S^n\times y_*\cup y_*\times S^n\subset S^n\times S^n,$$

as sub-spaces in the euclidean spaces of suitable dimensions.

Define the mapping $d_n: S^n\times I^1\to S^{n+1}$ as in [9, §1], which is characterized by the following properties:

$$d_n \text{ maps } (S^n-y_*)\times[0, 1) \text{ topologically on } E_+^n-y_*,$$
$$d_n \text{ maps } (S^n-y_*)\times(-1, 0] \text{ topologically on } E_-^n-y_*,$$
$$d_n(S^n\times \dot I^1\cup y_*\times I^1)=y_*, \text{ and } d_n(x, 0)=(x, 0).$$

We also define the mapping $\varphi_n: S^n\to S^n\vee S^n$ as in [9, §1], which maps subspaces $S_0^{n-1}=\{x\in S^n\,|\,x_2=0\}$ to the point $y_*\times y_*$ and elsewhere topologically preserving orientation.

We denote the point $(tx_1, \dots, tx_n)$ by tx, where $x=(x_1, \dots, x_n)$ and t is a real number.

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* Numbers in blackets refer to the references cited at the end of the paper.