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ON PRIME RIGHT IDEALS OF INTERMEDIATE RINGS OF A FINITE NORMALIZING EXTENSION

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Introduction

Throughout this paper, S will represent a ring extension of a ring R with common identity 1. Let I be a right ideal of R, and $b_R(I)=\{r\in R \mid Rr\subset I\}$. I is called a prime right ideal, provided that if X, Y are right ideals of R with $XY\subset I$, then either $X\subset I$ or $Y\subset I$. It is clear that every maximal right ideal is a prime right ideal. If I is a prime right ideal, then $b_R(I)$ is a prime ideal. Next, let R' be a ring. An R-R'-bimodule M is called a *torsionfree* R-R'-bimodule if $r_M(X)=l_M(Y)=0$ for every essential ideal X of R and every essential ideal Y of R', where $r_M(X)$ (resp. $l_M(Y)$) is the right (resp. left) annihilator of X (resp. Y) in M, and M is called a *finite normalizing* R-R'-bimodule if there exist elements $a_1, a_2, \cdots, a_n$ of M such that $M=\sum_{i=1}^n Ra_i$ and $Ra_i=a_iR'$ for $i=1, 2, \cdots, n$. Such a system $\{a_i\}_i$ is called a normailzing generating system of M. Finally S is a *finite normalizing extension* of R if S is a finite normalizing R-R-bimodule.

In [1], [2], [3], [4] and [6], "cutting down" theorems for a prime ideals were studied. In the previous paper [7], we have obtained a "cutting down" theorem for a prime right ideal of a finite normalizing extension under the hypothesis that the finite normalizing extension considered is torsionfree. The present objective is to reprove the same without the hypothesis "torsionfree"; we shall prove the following theorem.

Theorem. *Let S be an arbitrary finite normalizing extension of R, T a ring with $R\subset T\subset S$. If J is a prime right ideal of T, then there exist prime right ideals $K_1, K_2, \cdots, K_s$ of R such that $\cap_{i=1}^s K_i=J\cap R$. In this case, $b_R(J\cap R)=\cap_{i=1}^s b_R(K_i)$.*

1. Preliminaries

Throughout this paper, S will represent a finite normalizing extension of R, and T a ring with $R\subset T\subset S$.

Let P be a prime ideal of T. In studying P and T/P, one can usually reduce problems to the case in which (1) S is a prime ring, and (2) $A\cap T\not\subset P$ for