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ON PROJECTIVE MODULES OVER DIRECTLY FINITE REGULAR RINGS SATISFYING THE COMPARABILITY AXIOM II

Dedicated to Professor Hisao Tominaga on his 60th birthday

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In [4], by observing the directly finiteness of projective modules, the first author classified directly finite (d.f. for short) regular rings satisfying the comparability axiom (c. axiom for short) into three types: Type A, Type B and Type C.

In the present paper, we give a more explicite criterion of the directly finiteness of projective modules over each types and show the following for a d.f. regular ring R satisfying the c. axiom: (a) R is Type A if and only if $Soc(R)=0$ and the intersection $I_0(R)$ of all nonzero ideals of R is nonzero. (b) R is Type B if and only if $Soc(R)=0$, $I_0(R)=0$ and the family $L(R)$ of all ideals of R has a cofinal subfamily. (c) R is Type C if and only if $Soc(R)\neq 0$, or $I_0(R)=0$ and $L(R)$ does not have any cofinal subfamilies. As an application we show the following for a projective module P over a d.f. regular ring satisfying the c. axiom: P is directly infinite (d.inf. for short) if and only if P contains a direct summand which is isomorphic to $\aleph_0 X$ for a suitable nonzero module X.

Throughout this paper we assume that R is a d.f. regular ring satisfying the c. axiom, and all R-modules considered are unital right R-modules.

1. Notations and definitions

For two R-modules X and Y, we use $X\lesssim Y$ (resp. $X\lesssim\oplus Y$) to mean that X is isomorphic to a submodule of Y (resp. a direct summand of Y). $X\lnsim Y$ means that $X\lesssim Y$ and $X\not\cong Y$. For a submodule X of an R-module Y, $X<\oplus Y$ means that X is a direct summand of Y. For a cardinal number α and an R-module X, αX denotes a direct sum of α-copies of X. For a set I, we denote by $|I|$ the cardinal number of I. We denote by $L(R)$ the family of all ideals of R. Since R satisfies the c. axiom, $L(R)$ is a linearly ordered set under inclusion ([1, Proposition 8.5]). We put $I_0(R)=\cap\{I\mid 0\neq I\in L(R)\}$. We denote by $Soc(R)$ the socle of R. We note that if $Soc(R)\neq 0$ then it is homogeneous and coincides with $I_0(R)$.