Aida, S.
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ON THE IRREDUCIBILITY OF DIRICHLET FORMS ON DOMAINS IN INFINITE DIMENSIONAL SPACES

SHIGEKI AIDA

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1. Introduction

Let (B, H, μ) be an abstract Wiener space. Let $\phi : B \to \mathbb{R}$ be an $H-$continuous function and define $U := \{z \in B \mid \phi(z) > 0\}$. Assume $\mu(U) > 0$. Then U satisfies that for any $z \in U$, there exists an open set V_z in H such that $z + V_z \subset U$. Hence we may define an $H-$derivative for the function on U as in the Wiener space itself. In fact, Kusuoka [13, 14, 15] defined an $H-$derivative on U and define a Dirichlet form $\mathcal{E}_U$ on U and gave a criterion of the irreducibility of the Dirichlet form. Actually his assumption on U, namely $H-$connectivity and the regularity of ϕ, i.e., strong $C^\infty - C_0$ property implies a stronger property, uniform positivity improving property, (see [5] and Remark 11 in §2) than irreducibility. The author made use of his theorem to prove the irreducibility of the Dirichlet forms on loop spaces. The aim of this article is to prove the irreducibility of $\mathcal{E}_U$ without "strong $C^\infty - C_0$ property" and provide a simpler proof than Kusuoka's proof. Our proof does not use special properties of Gaussian measures and so our theorem may hold in more general situation (see Remark 10 in §2).

The organization of this paper is as follows. In §2, we will prove our main thorem and in §3, we will prove the irreducibility of the Dirichlet form on loop group using our method.

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2. Main Results

Let (B, H, μ) be an abstract Wiener space. Let a measurable function $\phi : B \to \mathbb{R}$ to be an $H-$continuous function, i.e. for any $z \in B$, $\phi(z + \cdot) : H \to \mathbb{R}$ is a continuous function. Let us set $U = \{z \in B \mid \phi(z) > 0\}$. Assume that $\mu(U) > 0$ and we denote $d\mu_U := d\mu|_U / \mu(U)$. Let us recall the definition of the Dirichlet form on U ([13]).