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EXTENSIONS OF SOME 2-GROUPS WHICH PRESERVE THE IRREDUCIBILITIES OF INDUCED CHARACTERS

Dedicated to Professor Yukio Tsushima on his 60th birthday

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1. Introduction

Let Q_n and D_n denote the generalized quaternion group and the dihedral group of order $2^{n+1}(n \geq 2)$, respectively. Let SD_n denote the semidihedral group of order $2^{n+1}(n \geq 3)$.

As is stated in [3], these groups have remarkable properties among all 2-groups.

Moreover, Yamada and Iida [4] proved the following interesting result:

Let $\mathbf{Q}$ denote the rational field. Let G be a 2-group and χ a complex irreducible character of G. Then there exist subgroups $H \rhd N$ in G and the complex irreducible character ϕ of H such that $\chi = \phi^G$, $\mathbf{Q}(\chi) = \mathbf{Q}(\phi)$, $N = \mathrm{Ker}\phi$ and

$$H/N \cong Q_n(n \geq 2),\ \text{or}\ D_n(n \geq 3),\ \text{or}\ SD_n(n \geq 3),\ \text{or}\ C_n(n \geq 0),$$

where C_n is the cyclic group of order 2^n, and $\mathbf{Q}(\chi) = \mathbf{Q}(\chi(g), g \in G)$.

In [3], Yamada and Iida considered the case when $N = 1$. Note that ϕ is faithful in this case. They studied the following problem:

Problem. Let ϕ be a faithful irreducible character of H, where $H = Q_n$ or D_n or SD_n. Determine the extension group G of H such that the induced character ϕ^G is also irreducible.

It is well-known that the groups Q_n, D_n and SD_n have faithful irreducible characters. It is also known that they are algebraically conjugate to each other. Hence the irreducibility of ϕ^G, where ϕ is a faithful irreducible character of Q_n or D_n or SD_n, is independent of the choice of ϕ, but depends only on these groups.

In [3], Yamada and Iida solved this problem when $[G : H] = 2$ and 4 for all $H = Q_n$ or D_n or SD_n.

The purpose of this paper is to solve this problem when $[G : H] = 8$ for all $H = Q_n$ or D_n or SD_n.