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SYMMETRIC UNIONS AND RIBBON KNOTS

CHRISTOPH LAMM

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1. Introduction

In their article "On unions of knots" [15] S. Kinoshita and H. Terasaka studied a way of connecting knot diagrams which generalizes the operation of connected sum: an additional two-string tangle with n half-twists is inserted between the two diagrams. For the case that the two knot diagrams are mirror images of each other they found that the Alexander polynomial depends only on the parity of n and that the determinant is independent of n.

The results of [15] generalize in a natural way to the case where several twist tangles are inserted. We call such a generalized union of a knot $\hat{K}$ with its mirror image $\hat{K}^*$ *symmetric union*, and $\hat{K}$ is called the *partial knot*.

In addition to results on the Alexander polynomial and the determinant, we use the homology of the double branched coverings and the knot groups to exclude possible partial knots for a given symmetric union. We prove that a symmetric union with non-trivial partial knot is itself non-trivial. (This is an analogue of the non-cancellation theorem for the connected sum of knots.)

Finally, we investigate the relationship between symmetric unions and ribbon knots. We succeed in finding symmetric diagrams for all but one of the 21 prime ribbon knots up to 10 crossings.

2. Symmetric unions

We denote the tangles made of half-twists by integers $n \in \mathbb{Z}$ and the horizontal trivial tangle by $\asymp$ (Fig. 1).

DEFINITION 2.1. Let D be an unoriented knot diagram and D^* the diagram D reflected at an axis in the plane. If in the symmetric placement of D and D^* we replace the tangles $T_i = 0$, $(i = 0, \ldots, k)$ on the symmetry axis by $T_i = \asymp$ for $i = 0, \ldots, \mu - 1$ and $T_i = n_i \in \mathbb{Z}$ for $i = \mu, \ldots, k$ (with $\mu \geq 1$), we call the result a *symmetric union* of D and D^* and write $D \cup D^*(T_0, \ldots, T_k)$. The *partial knot* $\hat{K}$ of the symmetric union is the knot given by the diagram D. See Fig. 1 for an illustration of the case $\mu = 1$.