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Osaka J. Math.
37 (2000), 441–451

FOUR-GENUS AND UNKNOTTING NUMBER OF POSITIVE KNOTS AND LINKS

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(Received June 16, 1998)

1. Introduction

A *link* is a closed 1-manifold embedded in the 3-sphere S^3 and a *knot* is a link with one connected component. The *unknotting number* of a knot K, denoted by $u(K)$, is the minimum number of crossing changes needed to create an unknotted diagram, where the minimum should be taken over all possible sets of changes in all possible diagrams representing K. The 4-*genus* (3-*genus* resp.) of a link L, denoted by $g^*(L)$ (by $g(L)$ resp.), is the minimum number of genera of all smooth compact connected and orientable surfaces bounded by $L \subset \partial B^4 = S^3$ in B^4 (in S^3 resp.). These are very intuitive invariants of knot and link types, but hard to compute for a given knot or link. L. Rudolph has studied the links which appear as the intersection of $S^3 = \{(z, w) \mid |z|^2 + |w|^2 = 1,\ z, w \in \mathbf{C}\}$ and an algebraic curve in the 2-dimensional complex plane $\mathbf{C}^2$ (cf. [10]). He obtained a formula for computing the 4-genera of *quasipositive links*, which are algebraic curves. In this paper, by using his results, we give a formula to compute directly the 4-genera of *positive links* from any positive diagrams.

Theorem 1.1. *Let L be a positive link with μ components, and D any non-split positive diagram of L. Then we have*

$$g(L) = g^*(L) = \frac{2 - \mu - s(D) + c(D)}{2},$$

where $s(D)$ is the number of Seifert circles, $c(D)$ is the number of crossings of D.

We remark that we can compute the genus of a splittable positive link from a split positive diagram by applying Theorem 1.1 to each connected component of diagrams and by summing up their genera.

In order to prove Theorem 1.1 we make a precise observation on Yamada's algorithm concerned with braid representations of knots and links, and apply Rudolph's theorem.

Using Theorem 1.1, we determine the positive knots with unknotting number one (Theorem 5.1). This gives an alternate proof of Przytycki-Taniyama's theorem [9].