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STOCHASTIC CALCULUS OF GENERALIZED DIRICHLET FORMS AND APPLICATIONS TO STOCHASTIC DIFFERENTIAL EQUATIONS IN INFINITE DIMENSIONS

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1. Introduction

In this paper we systematically develop, as a technical tool for our main application below, a stochastic calculus for *generalized* Dirichlet forms (cf. [15]). In particular, we show *Fukushima's decomposition* of additive functionals including its *extended version* to functions not necessarily in the domain $\mathcal{F}$ of the generalized Dirichlet form, cf. Theorem 4.5 (ii), and an *Itô-type formula* in this framework. The extended version of the Fukushima decomposition will be applied in combination with the Itô-type formula in Subsection 6.3. The class of generalized Dirichlet forms is much larger than the well-studied class of symmetric and coercive Dirichlet forms as in [6] resp. [10] and time dependent Dirichlet forms as in [12]. It contains examples of an entirely new kind (cf. Section 6, [15]). Therefore, the results obtained in this paper lead to extensions of the corresponding results in the "classical" theories. In particular the proofs are "locally" completely different (cf. e.g. Theorem 2.3 and Theorem 2.5; though for the reader's convenience we tried to follow the line of argument in [6] as closely as possible). This difference has several reasons: First of all we *do not assume any sector condition*; in certain cases we have to handle $\mathcal{E}$-quasi-lower-semicontinuous functions instead of $\mathcal{E}$-quasi-continuous functions (cf. e.g. Remark 2.6 (i)), and finally, the Dirichlet space of the classical situation generalizes to a space $\mathcal{F}$ which is not necessarily stable under composition with normal contractions. In contrast to the classical theory it is not known whether regularity or quasi-regularity alone imply the existence of an associated process. An additional structural assumption on $\mathcal{F}$ is made in [15, IV.2, D3] (i.e. the existence of a nice intermediate space $\mathcal{Y}$ has to be assumed) in order to construct explicitly an associated m-tight special standard process $\mathbf{M}$.

In addition to the new theoretical results described above we also present new applications. In Section 6 we construct weak solutions to stochastic differential equations in infinite dimensions of the type

$$dX_t = dW_t + \frac{1}{2}\beta_H^{\mu}(X_t)dt + \overline{\beta}(X_t)dt\,, \quad X_0 = z. \tag{1}$$