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ON THE GIBBS MEASURES OF COMMUTING ONE-SIDED SUBSHIFTS OF FINITE TYPE

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0. Introduction

This paper is concerned with the Gibbs measures on mixing one–sided subshifts of finite type. Let (X, S) be a mixing one–sided subshift of finite type and let $\varphi : X \to \mathbb{R}$ be a continuous function with summable variation. Then there exists a unique S–invariant probability measure $\mu_{S,\varphi}$ on X, called the *Gibbs measure* of the system (X, S, φ), which maximizes the measure theoretic pressure [6]. It is well known that if $\varphi, \psi : X \to \mathbb{R}$ are continuous functions on X with summable variation, then $\mu_{S,\varphi} = \mu_{S,\psi}$ if and only if there is a continuous function w on X such that

$$\partial_S\varphi - \partial_S\psi = \partial_S^2 w,$$

where the coboundary operator ∂_S is defined by $\partial_S f = f - f \circ S$ for any real-valued function f on X. In this case, if φ and ψ are Holder continuous, then w must be Holder continuous too. Moreover, it has recently been proved that if $T : X \to X$ is a positively expansive endomorphism and $S \circ T = T \circ S$, then (X, T) is also a mixing one–sided subshift of finite type [1, 3, 4, 5], and (X, S) and (X, T) have the same Parry measure, that is, $\mu_{S,0} = \mu_{T,0}$ [1, 3, 4]. In this paper, generalizing these results, we find a necessary and sufficient condition for two systems (X, S, φ) and (X, T, ψ) to have the same Gibbs measure(Theorem 2.2). Consequently, we prove that a cocycle admits an identical Gibbs measure(Theorem 2.3).

1. Preliminaries

Let us introduce some preliminaries. A dynamical system is a pair (X, S), where X is a compact metric space with metric d, and $S : X \to X$ is a continuous surjective mapping. A dynamical system (X, S) is called a *one–sided subshift* if there is a finite clopen partition $\mathcal{A}$, called an *alphabet* for (X, S), such that

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