Collin, O.
Osaka J. Math.
37 (2000), 57 – 71

EQUIVARIANT CASSON INVARIANT FOR KNOTS AND THE NEUMANN–WAHL FORMULA

OLIVIER COLLIN *

(Received May 20, 1998)

In 1985, Casson introduced a new 3-manifold invariant for integer homology spheres, via $SU(2)$–representation spaces of their fundamental groups. This invariant has solved some difficult problems in 3-manifold Topology and was found to have a relation to Gauge Theory, by work of Taubes in [17]. More recently, there have been various generalizations of Casson's invariant, for example Walker's extention to rational homology spheres. But more importantly for this article, in [3], Cappell, Lee and Miller have defined an equivariant Casson invariant, $\lambda^{(k/n)}(Y^3)$, for closed 3-manifolds Y^3 with a semi-free $\mathbb{Z}_n$–action, where n is an odd prime and k an integer such that $0 \leq k \leq (n-1)/2$. A particularly important case is that of n-fold cyclic branched coverings of S^3 along a knot K, denoted $V_n(K)$. In that case, the equivariant Casson invariants may be related to equivariant knot signatures of K, by developping a surgery formula for the invariants as done in [3].

In parallel, the author and B. Steer developped in [6] a Floer Homology for knots in S^3, a knot invariant consisting of four abelian groups, denoted $HF^{(\alpha)}(S^3, K)$, depending on a parameter $0 \leq \alpha \leq 1/2$. The Euler characteristic of this Floer Homology was related to equivariant knot signatures of K, via work of Herald in [9]. In fact, when $\alpha = k/n$ for n an odd prime, we have $\chi(HF_*^{(k/n)}(S^3, K)) = \lambda^{(k/n)}(V_n(K))$, so that the Floer Homology for knots may be seen as a generalisation of the equivariant Casson invariant of cyclic branched covers of S^3 along knots. In this article, we combine both points of view to define an equivariant Casson invariant for 3-manifolds which arise as 4-fold cyclic branched covers of S^3 along a knot K. In the case of cyclic branched covers with an even branching index, the construction in [3] does not apply. The original motivation for this work was therefore to complement the work done in [3]. The resulting invariant brings new light to the Neumann–Wahl formula, relating Casson's invariant of a 3-dimensional link of complex singularity to the signature of its Milnor fibre, as explained in Section 4, where the equivariant Casson invariant is expressed in terms of the Jones polynomial of K, using a result of Mullins. The article ends with a generalization to other cyclic branched coverings of S^3 with

*The author is supported by an NSERC Postdoctoral Fellowship.