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SHORT PROOFS OF HIRAMINE' RESULTS ON CHARACTER VALUES

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1. Introduction

Let G and H be finite groups of order n. A mapping f from G into H is called a planar function of degree n if, for each element $\tau \in H$ and $u \in G^* = G - \{1\}$, there exists exactly one $x \in G$ such that $f(ux)f(x)^{-1} = \tau$. In [2] Hiramine has shown that if both G and H are abelian groups of order $3p$ with $p(\geq 5)$ a prime, then there exists no planar function from G into H. To prove this he has established two results on character values. Their proofs are slightly complicated. In this note we shall give short proofs.

In section 2 we shall present Proposition 2 which is useful for the proof of Result 2. In section 3 we shall state Hiramine' results and give short proofs.

We follow the notation and terminology of [2].

2. Planar Functions and Equations in Group Algebras

Let G and H be finite groups of order n. Throughout this article elements of G will be denoted by small Roman letters and elements of H by small Greek letters.
Let f be a mapping from G into H and $S_\alpha = \{x \in G | f(x) = \alpha\}$, $\alpha \in H$. If $S_\alpha \neq \emptyset$, we set $\hat{S_\alpha} = \sum_{x \in S_\alpha} x \in C[G]$ and $\hat{S}_\alpha^{-1} = \sum_{x \in S_\alpha} x^{-1} \in C[G]$, otherwise $\hat{S_\alpha} = \hat{S}_\alpha^{-1} = 0$, where $C[G]$ is the group algebra of G over the complex number field C. Let $G_0 = G \times H$ be the direct product of groups G, H.

To prove the results we need two propositions.

The following is Proposition 2.1 [2].

Proposition 1. *The following are equivalent.*

(i) *The function f is planar.*

(ii) *In the group algebra $C[G]$ of G,*

$$\sum_{\alpha \in H} \hat{S}_{\tau\alpha} \hat{S}_\alpha^{-1} = \sum_{\alpha \in H} \hat{S}_{\alpha\tau}^{-1} \hat{S}_\alpha = \begin{cases} \hat{G} + n - 1 & \textit{if } \tau = 1, \\ \hat{G} - 1 & \textit{otherwise.} \end{cases}$$