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EXISTENCE OF PENCILS WITH PRESCRIBED SCROLLAR INVARIANTS OF SOME GENERAL TYPE

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0. Introduction

Let C be an irreducible smooth projective non-hyperelliptic curve of genus g defined over the field $\boldsymbol{C}$ of complex numbers. Let g_k^1 be a complete base-point free special linear system on C. The scrollar invariants of g_k^1 are defined as follows. Let C be canonically embedded in $\boldsymbol{P}^{g-1}$ and let X be the union of the linear spans $\langle D\rangle$ with $D \in g_k^1$. This defines a set of integers $e_1 \geq \ldots \geq e_{k-1} \geq 0$ such that X is the image of the projective bundle $\boldsymbol{P}(e_1; \ldots ; e_{k-1}) = \boldsymbol{P}(O_{P^1}(e_1) \oplus \ldots \oplus O_{P^1}(e_{k-1}))$ using the tautological bundle (see e.g. [2]; [7]). Those integers $e_1; e_2; \ldots ; e_{k-1}$ are called the scrollar invariants of g_k^1.

Those scrollar invariants determine (and are determined by) the complete linear systems associated to multiples of the linear system g_k^1. For $1 \leq i \leq k-1$ the invariant e_i is one less than the number of non-negative integers j satisfying $\dim(|K_C - jg_k^1|) - \dim(|K_C - (j+1)g_k^1|) \geq i$. Here K_C denotes a canonical divisor on C. Let $m = e_{k-1}+2$. Then m is defined by the following conditions: $\dim(|(m-1)g_k^1|) = m-1$ and $\dim(|mg_k^1|) > m$. In case $|mg_k^1|$ is birationally very ample then the scrollar invariants satisfy the inequalities $e_i \leq e_{i+1} + m$ for $1 \leq i \leq k-2$ (see [3]). In case $k = 3$ this number $m = e_2$ determines also the other scrollar invariant e_1. It is the starting point for so-called Maroni-theory for linear systems on trigonal curves (see [4]; [5]). Scrollar invariants for 4-gonal curves are intensively studied in [1]; [3] and for 5-gonal curves in [6].

For $(j-1)m - 1 < x \leq jm - 1$ with $j \leq k-1$ the inequalities between the scrollar invariants imply $\dim(|xg_k^1|) \geq \frac{j(j-1)}{2}m - 1 + (x - (j-1)m + 1)j$. Equality (if not in conflict with the Riemann-Roch Theorem) can be expected being the most general case for a fixed value of m. The inequalities also imply $\dim(|(k-1)mg_k^1|) = \dim(|((k-1)m-1)g_k^1|) + k$. This implies that $|((k-1)m-1)g_k^1|$ is not special. Using the dimension bound one obtains $g \leq [(k^2-k)m - 2k + 2]/2$. (This easy but interesting consequence from the inequalities is not mentioned by Kato and Ohbuchi.) In this paper we prove the following theorem.

Theorem. *For all nonnegative integers k; m and g satisfying $k \geq 3$; $m \geq 2$ and $k-1 \leq g \leq [(k^2-k)m-2k+2]/2$ there exists a smooth curve C of genus g possessing*