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L^q-SPECTRUM OF BERNOULLI CONVOLUTIONS ASSOCIATED WITH P.V. NUMBERS

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1. Introduction

Let μ be a positive bounded Borel measure on $\mathbb{R}^d$ with bounded support and let supp(μ) denote the support of μ. For $\delta > 0$ and $q \in \mathbb{R}$, the *L^q-(moment) spectrum* of μ is defined as

$$\tau(q) = \varliminf_{\delta \to 0^+} \frac{\ln \sup \sum_i \mu(B_\delta(x_i))^q}{\ln \delta},$$

where $\{B_\delta(x_i)\}_i$ is a disjoint family of δ-balls with center $x_i \in \text{supp}(\mu)$ and the supremum is taken over all such families. For $q > 1$, the *L^q-dimension* (or *generalized Rényi dimension*, see e.g. [10], [22]) of μ is defined as

$$\underline{\dim}_q(\mu) = \frac{\tau(q)}{q-1}.$$

The spectra $\tau(q)$ and $\underline{\dim}_q(\mu)$ play a central role in studying the multifractal structure of the measure μ (e.g., the multifractal formalism [6], [9], [10]) and it is of great interest to compute them. There is a simple formula for $\tau(q)$ if μ is a self-similar measure defined by an iterated function system of contractive similitudes satisfying the *open set condition* (OSC) ([3], [4], [8], [18], [19]).

The OSC is a separation condition on the similitudes. In the absence of this condition, the dynamics of the iteration is not clear and very few results are known. In [14] the authors introduced a weak separation condition to study some interesting self-similar measures defined by similitudes that do not satisfy the OSC. An important class of examples comes from the self-similar measure μ satisfying the identity

$$\mu = \frac{1}{2}\mu \circ \psi_1^{-1} + \frac{1}{2}\mu \circ \psi_2^{-1}, \tag{1.1}$$

where $\psi_1 x = \rho x$, $\psi_2 x = \rho x + (1-\rho)$, $1/2 < \rho < 1$ ([12], [13], [14]). It is called an *infinitely convolved Bernoulli measure* (ICBM) because it can be identified (up to a scalar

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