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ON PROBABILISTIC APPROACH TO THE EIGENVALUE PROBLEM FOR MAXIMAL ELLIPTIC OPERATOR

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0. Introduction

Let D be a bounded domain of R^d with its smooth boundary ∂D. Let $L^\alpha, \alpha \in A$ (A is a given set), be non-degenerate second-order linear differential operator with parameter α of the following type:

$$L^\alpha = \sum_{i,j=1}^{d} a_{ij}(\alpha, x)\partial_i\partial_j + \sum_{i=1}^{d} b_i(\alpha, x)\partial_i - c(\alpha, x) \tag{0.1}$$

and let also L be a (nonlinear) operator, given by the formula $L = \sup_{\alpha \in A} L^\alpha$, which is called the maximal operator. Let consider a Dirichlet eigenvalue problem with respect to L on D;

$$Lu + \lambda u = 0 \text{ on } D,\ u > 0 \text{ on } D, \text{ and } u|_{\partial D} = 0. \tag{0.2}$$

In this paper, under the assumption that there exist an eigenvalue λ and corresponding (smooth) eigenfunction u satisfying (0.2), we will discuss various properties of λ and also obtain a probabilistic representation for λ.

We shall prove in §2 that λ is smaller than any λ^α, $\alpha \in A$, where for each $\alpha \in A$, λ^α is the smallest eigenvalue of linear operator L^α. In §3, we shall show that λ is the limit of a sequence of the principal eigenvalues, each of them corresponds to a linear operator in the class. Finally, in §4, we shall obtain a probabilistic representation for λ.

Our method of proof is similar to that of [3] and [4]. Namely, it proceeds firstly by considering the transformation such that $v = -\log u$, after then by applying Bellman equation's method to the equation with respet to v obtained by such logarithmic transformation of (0.2). Therefore our results mainly rely on the theory of stochastic control and Bellman equation developped by [1], [5] and [6](See also [2]).

C.Pucci showed in [8] that $\lambda = \min \lambda^\alpha$, in the case where $a(\alpha, x) = \alpha(x)$ and $\alpha(\cdot)$ is a matrix-valued bounded measurable function. In [8], he also gave an interesting example in which $D = \{x \in R^d; |x| < 1\}$, and there exist an eigenvalue and corresponding (smooth)