Nagase, M. and Yoshida, M.
Osaka J. Math.
36 (1999), 919–933

COMMUTATOR ESTIMATES AND A SHARP FORM OF GÅRDING'S INEQUALITY

MICHIHIRO NAGASE AND MANABU YOSHIDA

(Received November 28, 1997)

0. Introduction

In the present paper we show a commutator estimate of pseudo-differential operators in the framework of $L^2(\mathbb{R}^n)$. As an application we give a sharp form of Gårding's inequality for sesqui-linear forms with coeffiencients in $\mathcal{B}^2$. There has been similar kinds of commutator estimates. In [5], Kumano-go and Nagase obtain a result on commutator estimates and used it to show a sharp form of Gårding's inequality for sesqui-linear form defined by elliptic differential operators of the form

$$B[u,v] = \sum_{|\alpha|\leq m, |\beta|\leq m} (a_{\alpha\beta}(x) D_x^\alpha u, D_x^\beta v)$$

where the coefficients $a_{\alpha\beta}(x)$ are $\mathcal{B}^2(\mathbb{R}^n)$ functions.

In [3], Koshiba shows a sharp form of Gårding's inequality for the form

$$B[u,v] = (p(X,D_x)u, v)$$

where the symbol $p(x,\xi)$ of the operator $p(X,D_x)$ is $\mathcal{B}^2$ smooth in space variable x and homogeneous in covariable ξ, and used the sharp form of Gårding's inequality to the study of the stability of difference schemes for hyperbolic initial problems. On the other hand in [2], N. Jacob shows Gårding's inequality for the form

$$B[u,v] = \sum_{i,j=1}^{m} \int_{\mathbb{R}^n} \overline{a_{i,j}(x) Q_j(D) u(x)} P_i(D) v(x) dx$$

where $P_i(D)$ and $Q_j(D)$ are pseudo-differential operators, and $a_{i,j}(x)$ are non-smooth functions. The symbol class of the present paper is similar to the one in [2].

In section 1, as a preliminary we give definitions and fundamental facts of pseudo-differential operators. In section 2 we treat commutator estimates and give the main theorem relative to the commutator estimate. Finally in section 3 we give the sharp form of Gårding's inequalities for our class of operators.