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SCATTERING THEORY FOR TIME-DEPENDENT HARTREE-FOCK TYPE EQUATION

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1. Introduction

In this paper we consider the scattering problem for the following system of nonlinear Schrödinger equations with nonlocal interaction

$$
\begin{aligned}
&(1) \qquad & i\frac{\partial}{\partial t}u_j &= -\frac{1}{2}\Delta u_j + f_j(\vec{u}), \quad (t,x)\in \mathbf{R}\times\mathbf{R}^n,\\
&(2) \qquad & u_j(0,x) &= \phi_j(x), \qquad j=1,\cdots,N.
\end{aligned}
$$

Here Δ denotes the Laplacian in x,

$$f_j(\vec{u}) = \sum_{k=1}^{N}(V*|u_k|^2)u_j - \sum_{k=1}^{N}[V*(u_j\bar{u}_k)]u_k,$$

and $*$ denotes the convolution in $\mathbf{R}^n$. In this paper we treat the case $n\geq 2$ and $V(x)=|x|^{-\gamma}$ with $0<\gamma<n$.

The system (1)-(2) appears in the quantum mechanics as an approximation to a fermionic N-body system and is called the time-dependent Hartree-Fock type equation.

Throughout the paper we use the following notation:

$\mathbf{N}=\{1,2,3,\cdots\}$, $\nabla=(\partial/\partial x_1,\cdots,\partial/\partial x_n)$, $U(t)=\exp(it\Delta/2), M(t)=\exp(i|x|^2/2t)$, $J=U(t)xU(-t)=M(t)(it\nabla)M(-t)$. For $1\leq p\leq\infty$, $p'=p/(p-1)$, $\delta(p)=n/2-n/p$. $\|\cdot\|_p$ denotes the norm of $L^p(\mathbf{R}^n)$ (if $p=2$, we write $\|\cdot\|_2=\|\cdot\|$). For $1\leq q,r\leq\infty$ and for the interval $I\subset\mathbf{R}$, $\|\cdot\|_{q,r,I}$ denotes the norm of $L^r(I;L^q(\mathbf{R}^n))$, namely, $\|u\|_{q,r,I}=\left[\int_I\left(\int_{\mathbf{R}^n}|u(t,x)|^q dx\right)^{r/q}dt\right]^{1/r}$. For positive integers l and m, $\Sigma^{l,m}$ denotes the Hilbert space defined as

$$\Sigma^{l,m}=\left\{\psi\in L^2(\mathbf{R}^n); \|\psi\|_{\Sigma^{l,m}}=\Big(\sum_{|\alpha|\leq l}\|\nabla^\alpha\psi\|^2+\sum_{|\beta|\leq m}\|x^\beta\psi\|^2\Big)^{1/2}<\infty\right\}.$$

When we use N'th direct sums of various function spaces, we denote them by the same symbols and denote these elements by writing arrow over the letter, like $\vec{f}$.

Now we state our main theorem.