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RIEMANNIAN G.O. SPACES FIBERED OVER IRREDUCIBLE SYMMETRIC SPACES

Dedicated to Professor Tadashi Nagano on his 70th birthday

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0. Introduction

We will classify the homogeneous spaces $M = G/K$ satisfying that (i) M are fibered over irreducible symmetric spaces G/H, and (ii) certain G-invariant metrics on G/K are g.o. with respect to G. Among them are many examples of weakly symmetric spaces, which were described in [4], [25] and [5]. In fact we will obtain new examples of weakly symmetric spaces. We will find very useful the theory of orbits of s-representations ([10] and [21]), and the classification of real irreducible polar representations ([6]).

A connected Riemannian manifold is called *g.o.* (geodesic orbit) if every geodesic is an orbit of a one-parameter subgroup of the isometry group. Every symmetric space is g.o. A connected Riemannian manifold is called *weakly symmetric* if, for any two points, there exists an isometry which interchanges them. Weakly symmetric spaces are also g.o. ([2]).

To study these spaces (more generally, Riemannian homogeneous spaces), the isotropy representations will play important roles. For example, W. Ziller characterized weakly symmetric spaces in terms of the isotropy representations, and provided many examples ([25]).

The isotropy representations of semi-simple symmetric spaces are called *s-representations*. One of their most interesting properties is the conjugacy of maximal abelian subspaces (see Lemma 1.5). A representation of a compact Lie group, which has the above property, is called *polar* (see below for exact definition). J. Dadok has investigated them, and classified irreducible ones ([6]). It is natural to expect that there are some relations between symmetric-like Riemannian manifolds and generalizations of s-representations.

In [7] and [20], g.o. metrics on a compact fiber bundle

$$F := H/K \longrightarrow M := G/K \longrightarrow B := G/H$$

have been investigated. A necessary and sufficient condition for certain metrics to be g.o., can be expressed in terms of the Lie algebras. Actually, the assumption that G is