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Quasi KO_*-types of CW-spectra X with $KU_*X \cong Free \oplus Z/2^m$

Dedicated to the memory of Professor Katsuo Kawakubo

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1. Introduction

Let KO, KU and KC denote the real, the complex and the self-conjugate K-spectrum, respectively. Given CW-spectra X, Y we say that X is quasi KO_*-equivalent to Y if $KO \wedge X$ is isomorphic to $KO \wedge Y$ as a KO-module spectrum, in other words, if there exists a map $h : Y \to KO \wedge X$ inducing an isomorphism $h_* : KO_*Y \to KO_*X$. Note that if X is quasi KO_*-equivalent to Y, then KU_*X is isomorphic to KU_*Y as a ($Z/2$-graded) abelian group with involution ψ_C^{-1}, in this case we say that X has the same C-type as Y. We are interested in the determination of the quasi KO_*-type of any CW-spectrum X using the information of its KU-homology group $KU_*X \cong KU_0X \oplus KU_1X$ with the conjugation ψ_C^{-1}.

Let $\eta : \Sigma^1 \to \Sigma^0$ be the stable Hopf map of order 2 and $C(\eta^l)$ denote the cofiber of the map $\eta^l : \Sigma^l \to \Sigma^0$. The sphere spectrum $S = \Sigma^0$ and the cofibers $C(\eta^l)$ $(l = 1, 2)$ are typical examples of spectra X with KU_*X free. In [1, Theorem 3.2] Bousfield has completely determined the quasi KO_*-type of a CW-spectrum X with KU_*X free.

Bousfield's Theorem . *Let X be a CW-spectrum such that $KU_*X \cong KU_0X \oplus KU_1X$ is free. Then it has the same quasi KO_*-type as a certain wedge sum of copies of $\Sigma^i(0 \le i \le 7), \Sigma^j C(\eta)(0 \le j \le 1)$ and $\Sigma^k C(\eta^2)(0 \le k \le 3)$.* (Cf. [6, Theorem 2.4]).

Let $SZ/2^m$ denote the Moore spectrum of type $Z/2^m$. In [4] and [5] we introduced some 3-cells spectra X_m and X'_m constructed as the cofibers of certain maps $f : \Sigma^i \to SZ/2^m$ and $f' : \Sigma^{i-1}SZ/2^m \to \Sigma^0$ and some 4-cells spectra $XY_m, X'Y'_m$ and $Y'X_m$ obtained as the cofibers of their mixed maps. In [5, Theorems 3.3, 4.2 and 4.4] by using these small spectra we have also determined the quasi KO_*-type of a CW-spectrum X such that $KU_0X \cong F \oplus Z/2^m$ with F free and $KU_1X = 0$. The purpose of this note is to determine completely the quasi KO_*-type of a CW-spectrum X such that $KU_*X \cong F \oplus Z/2^m$ with F free and finitely generated, without