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LARGE DEVIATIONS AND RELATED LIL'S FOR BROWNIAN MOTIONS ON NESTED FRACTALS

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1. Introduction

In [11], Donsker and Varadhan applied their celebrated large deviation theory for general Markov processes to the symmetric stable process $X(t)$ on $\boldsymbol{R}^1$ of index α, $0 < \alpha \leq 2$, and, by making use of the scaling property of $X(t)$, they proved that the accumulation points of scale changed occupation time distributions

$$\hat{L}_t(\omega, \cdot) = \frac{1}{t}\int_0^t \chi_\cdot\left(\left(\frac{\log\log t}{t}\right)^{1/\alpha} X(s)\right) ds \tag{1.1}$$

as $t \to \infty$ in the space $\mathcal{M}$ of subprobability measures on $\boldsymbol{R}^1$ endowed with the vague topology coincide almost surely with its subspace

$$C = \{\beta \in \mathcal{M} : I(\beta) \leq 1\}, \tag{1.2}$$

where $I(\beta)$ denotes the I-function in the large deviation principle.

From this, they deduced, among other things, the "other" law of the iterated logarithm

$$\liminf_{t\to\infty}\left(\frac{\log\log t}{t}\right)^{1/\alpha} \sup_{0\leq s\leq t} |X(s)| = \ell_\alpha(>0)\ a.s. \tag{1.3}$$

and a LIL for the local time (in case $d = 1, 1 < \alpha \leq 2$),

$$\limsup_{t\to\infty}\left(\frac{t}{\log\log t}\right)^{1/\alpha} \frac{1}{t}\sup_y \ell_t(\omega, y) = d_\alpha\ a.s. \tag{1.4}$$

extending the older results for the Brownian motion (the case that $\alpha = 2$) due to Chung [4], Jain and Pruitt [21], and Kesten [22]. As compared to the ordinary law of the

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