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THE QUASI KO_*-TYPES OF WEIGHTED MOD 4 LENS SPACES

Dedicated to Professor Fuichi Uchida on his sixtieth birthday

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0. Introduction

Let KU and KO be the complex and the real K-spectrum, respectively. For any CW-spectrum X its KU-homology group KU_*X is regarded as a ($Z/2$-graded) abelian group with involution because KU possesses the conjugation ψ_C^{-1}. Given CW-spectra X and Y we say that X is quasi KO_*-equivalent to Y if there exists an equivalence $f : KO \wedge X \to KO \wedge Y$ of KO-module spectra (see [8]). If X is quasi KO_*-equivalent to Y, then KO_*X is isomorphic to KO_*Y as a KO_*-module, and in addition KU_*X is isomorphic to KU_*Y as an abelian group with involution. In the latter case we say that X has the same $\mathcal{C}$-type as Y (cf. [2]). In [10] and [11] we have determined the quasi KO_*-types of the real projective space RP^k and its stunted projective space RP^k/RP^l. Moreover in [12] we have determined the quasi KO_*-types of the mod 4 lens space L_4^k and its stunted lens space L_4^k/L_4^l where we simply denote by L_4^{2n+1} the usual $(2n+1)$-dimensional mod 4 lens space $L^n(4)$ and by L_4^{2n} its $2n$-skeleton $L_0^n(4)$. In this note we shall generally determine the quasi KO_*-types of a weighted mod 4 lens space $L^n(4; q_0, \cdots, q_n)$ and its $2n$-skeleton $L_0^n(4; q_0, \cdots, q_n)$ along the line of [12].

The weighted mod 4 lens space $L^n(4; q_0, \cdots, q_n)$ is obtained as the fiber of the canonical inclusion $i : P^n(q_0, \cdots, q_n) \to P^{n+1}(4, q_0, \cdots, q_n)$ of weighted projective spaces (see [3]). Using the result of Amrani [1, Theorem 3.1] we can calculate the KU-cohomology group $KU^*L^n(4; q_0, \cdots, q_n)$ and the behavior of the conjugation ψ_C^{-1} on it. Our calculation asserts that $\Sigma^1 L_0^n(4; q_0, \cdots, q_n)$ has the same $\mathcal{C}$-type as one of the small spectra $\Sigma^2 SZ/2^r \vee P'_{s,t}$, $SZ/2^r \vee P''_{s,t}$ and $PP'_{r,s,t}$, and $\Sigma^1 L^n(4; q_0, \cdots, q_n)$ has the same $\mathcal{C}$-type as one of the small spectra $\Sigma^2 M_r \vee P'_{s,t}$, $M_r \vee P''_{s,t}$, $MPP'_{r,s,t}$ and $\Sigma^{2m} \vee \Sigma^1 L_0^n(4; q_0, \cdots, q_n)$ (see Proposition 3.2). Here $SZ/2^r$ is the Moore spectrum of type $Z/2^r$ and M_r, $P'_{s,t}$, $P''_{s,t}$, $PP'_{r,s,t}$ and $MPP'_{r,s,t}$ are the small spectra constructed as the cofibers of the maps $i\eta : \Sigma^1 \to SZ/2^r$, $i\bar{\eta} : \Sigma^1 SZ/2^t \to SZ/2^s$, $i\bar{\eta} + \tilde{\eta}j : \Sigma^1 SZ/2^t \to SZ/2^s$, $(\tilde{\eta}j, i\bar{\eta}) : \Sigma^1 SZ/2^t \to SZ/2^r \vee SZ/2^s$ and $(i_M\tilde{\eta}j, i\bar{\eta}) : \Sigma^1 SZ/2^t \to M_r \vee SZ/2^s$, respectively, in which $i : \Sigma^0 \to SZ/2^r$ and $j : SZ/2^r \to \Sigma^1$ are the bottom cell inclusion and the top cell