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Osaka J. Math.
35 (1998), 873–893

INTERSECTION NUMBERS FOR LOGARITHMIC *K*-FORMS

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(Received May 21, 1997)

1. Introduction

Let $L_j : \ell_j = 0$ $(0 \leq j \leq n+1)$ be hyperplanes in the k-dimensional projective space $\mathbf{P}^k$ in general position. For

$$\alpha = (\alpha_0, \dots, \alpha_{n+1}), \quad \alpha_j \in \mathbf{C} \setminus \mathbf{Z}, \quad \sum_{j=0}^{n+1} \alpha_j = 0,$$

we consider the logarithmic 1-from

$$\omega = \sum_{j=0}^{n+1} \alpha_j \frac{d\ell_j}{\ell_j}$$

and the covariant derivation $\nabla_\omega = d + \omega\wedge$ with respect to ω on $X = \mathbf{P}^k \setminus \cup_{j=0}^{n+1} L_j$. The k-th twisted de Rham cohomology group on X with respect to ω and that with compact support are defined as

$$H^k(X, \nabla_\omega) = \{\xi \in \mathcal{E}^k(X) \mid \nabla_\omega \xi = 0\}/\nabla_\omega \mathcal{E}^{k-1}(X),$$
$$H_c^k(X, \nabla_\omega) = \{\xi \in \mathcal{E}_c^k(X) \mid \nabla_\omega \xi = 0\}/\nabla_\omega \mathcal{E}_c^{k-1}(X),$$

where $\mathcal{E}^m(X)$ is the space of smooth m-forms on X and $\mathcal{E}_c^m(X)$ is the space of s-mooth m-forms on X with compact support. It is known that the inclusion of $\mathcal{E}_c^m(X)$ in $\mathcal{E}^m(X)$ induces a natural isomorphism of $H_c^k(X, \nabla_\omega)$ onto $H^k(X, \nabla_\omega)$ and that $H^k(X, \nabla_\omega)$ is spanned by

$$\varphi_I = d\log\left(\frac{\ell_{i_0}}{\ell_{i_1}}\right) \wedge d\log\left(\frac{\ell_{i_1}}{\ell_{i_2}}\right) \wedge \dots \wedge d\log\left(\frac{\ell_{i_{k-1}}}{\ell_{i_k}}\right),$$
$$I = (i_0, i_1, \dots, i_k), \quad 0 \leq i_0 < i_1 < \dots < i_k \leq n+1.$$

The groups $H_c^k(X, \nabla_\omega)$ and $H^k(X, \nabla_{-\omega})$ for $-\alpha = (-\alpha_0, \dots, -\alpha_{n+1})$ are dual to each other under the pairing